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MACHINE INTELLIGENCE AND THREAT IDENTIFICATION SYSTEMS

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This report presents the results of a study of various evidential reasoning paradigms. The paradigms were evaluated for their efficiency and effectiveness in solving classification problems as they apply to electronic warfare threat identification. The paradigms tested were Bayesian networks, the Dempster-Shafer theory of evidential reasoning, and fuzzy evidential reasoning. Domain knowledge and evidence are discussed as they apply to evidential reasoning systems. The support generation techniques for each of the evidential reasoning paradigms is described. The mathematical computations needed to construct the measure of support are defined and illustrated. A Generic Classification Tool (GCT) is presented with an overview of the data structures, design, features, and architecture. The methodology employed and the results of the analysis of the reasoning paradigms is presented The domain knowledge bases and evidence used in the analysis are described. The performance of the alternative support generation paradigms over a range of imprecise domain knowledge and information are compared.

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1 Introduction

The objective of this project is to analyze the effectiveness and efficiency of various evidential reasoning paradigms. The class of problems considered, referred to as *classification problems*, obtain and process information to identify an object from a set of possibilities. The information utilized in classification problems is often obtained from mechanical sensors or human observations and thus is inherently imprecise. The information is analyzed and used to produce measure of support that estimates the likelihood of each possibility based on the accumulated evidence. Classification problems have proved to be one of the most active areas of artificial intelligence research. Applications include diagnostic expert systems, decision analysis, and identification problems. The motivating application of this project is the identification of radar types based on information obtained through passive sensors.

The initial phase of the project was to examine techniques commonly employed in artificial intelligence (AI) applications for reasoning in domains in which the available information is incomplete and/or imprecise. The objective was to identify theoretically sound reasoning paradigms and architectures for evidence driven classification problems. The results of this study were presented in [9]. The mathematical paradigms selected for further analysis were probabilistic reasoning using Bayesian networks, the Dempster-Shafer theory of evidential reasoning, and fuzzy evidential reasoning.

The second phase was to develop a tool for comparing the effectiveness of the techniques identified in the initial analysis. During this phase, techniques and data structures were selected for the representation of domain knowledge and the interpretation of sensor data. This was followed by the design and construction of a software tool to compare the effectiveness of the various reasoning techniques. The Generic Classification Tool (GCT) was implemented in Common Lisp on a SUN-4 Workstation. An overview of the data structures, design, and features of the GCT is given in Sections 2–4. A more thorough presentation of these topics can be found in [8].

Section 2 reviews the knowledge representation techniques utilized in the GCT. An evidential reasoning system requires representations for two complementary types of problem specific knowledge: domain knowledge and evidence. Domain knowledge consists of the information that describes and distinguishes the alternatives. A frame-based knowledge representation schema was chosen to represent domain information. The characteristics of the objects are represented using partial membership functions, the mathematical basis of fuzzy set theory. The information provided by the sensors for the identification must be transformed into a format to be analyzed by the GCT. Possibility distributions, the fuzzy analog of conditional probability distributions, are used to represent imprecise sensor data.

The knowledge representation techniques provide the foundation for the analysis

of the likelihood of an object based on acquired information. Section 3 describes the support generation techniques for each of the evidential reasoning paradigms. The mathematical computations needed to construct the measure of support are defined and illustrated through several examples. The representation techniques and computational strategies described in Sections 2 and 3 are incorporated in the GCT. The features and architecture of the GCT are presented in Section 4.

Finally, the methodology employed and results of the analysis of the reasoning paradigms are presented. The domain knowledge bases and the evidence used in the analysis are described in Section 5. Section 6 compares the performance of the alternative support generation paradigms over a range of imprecise domain knowledge and information.

2 Knowledge Representation

In this section we review the representation of domain knowledge and evidential information. Fuzzy set theory and possibility theory were introduced by Zadeh [11] as a mathematical formalism for representing partial set-theoretic membership. Partial membership provides a sound mathematical foundation for representing imprecise and uncertain information. Fuzzy set theory was chosen as the basis for the representation of uncertainty because of its flexibility and the ability to transform fuzzy representations into those required by probabilistic systems and Dempster-Shafer systems. Section 2.1 introduces the formal techniques used to represent the objects in the frame of discernment. Domain knowledge is represented using a frame based structure similar to that developed by Zadeh [13,14] for encapsulating imprecise and incomplete domain knowledge in fuzzy relational data base systems. The GCT representation of domain knowledge is presented in Section 4.2.

The second type of information that must be formalized is the sensor data that drives the identification process. Possibility distributions, the fuzzy analog of conditional probability distributions, provide the basis for the representation of sensor data. Section 2.2 describes the relationship between observations and their mathematical interpretation as possibility distributions. Throughout this section, the standard fuzzy set theoretic notation for membership functions and possibility distributions are employed (see, for example, [2,8]).

2.1 Domain Knowledge Representation

The set of domain objects in a classification problem is known as the *frame of discernment.* The characteristics that describe and distinguish the elements in the frame of discernment represent the domain knowledge of a classification problem. This information is represented using an ***attribute-value*** representation. Attributes specify general characteristics or features of an object. Associated with each attribute is a finite set consisting of the values that the attribute may assume. The attributes provide the *frame* structure that will be used to define each of the domain objects.

Domain knowledge is represented using an attribute-value format in which the properties of the objects are represented by partial membership functions. If domain objects are characterized by attributes $at_1, \ldots, at_k$, the frame for the objects in the domain knowledge base has the form

$$\begin{array}{l} at_1 : \underline{\hspace{3cm}} \\ at_2 : \underline{\hspace{3cm}} \\ \quad \vdots \\ at_k : \underline{\hspace{3cm}} \end{array}$$

A particular domain object h is defined by instantiating each of the slots in the frame with a partial membership function that describes the characteristics of the corresponding attribute.

$$\begin{aligned} h : &\widetilde{at}_1(h) \\ &\widetilde{at}_2(h) \\ &\vdots \\ &\widetilde{at}_k(h). \end{aligned}$$

For an object h, the partial membership function $\widetilde{at}_i(h)$ specifies the consistency and the compatibility of the values of the attribute at_i with h. The domain of $\widetilde{at}_i(h)$ is the set V_i of possible values of the attribute at_i. A partial membership function for an attribute at_i with domain $V_i = \{v_1, \ldots, v_n\}$ is denoted

$$\widetilde{at}_i(h) = x_1/v_1 + \ldots + x_j/v_j + \ldots + x_n/v_n$$

where $x_j \in [0,1]$. The magnitude of x_j indicates the degree of compatibility of value v_j with the domain information describing h. If v_j is assigned 1, then it is completely compatible with the information describing the properties of h. Assigning 0 to v_j indicates that v_j is inconsistent with the domain information describing attribute at_i. The *focal set* of a partial membership function is the subset of the domain whose elements have membership value greater than 0. The focal set of $\widetilde{at}_i(h)$ consists of precisely the values that are consistent with the object h.

The partial membership function defined by the fuzzy set $\widetilde{at}_i(h)$ is often written using the functional notation

$$\mu_{at_i(h)}(v_j) = x_j.$$

The use of partial membership functions permits a great deal of flexibility in representing domain information. When the value of an attribute at_i is known to be v_j with complete certainty for object h, the at_i slot in the frame for h is instantiated with the membership function

$$\mu_{at_i(h)}(x) = \begin{cases} 1 & \text{if } x = v_j \\ 0 & \text{otherwise.} \end{cases}$$

The membership function that assigns 1 to each of the elements in the associated set V represents complete ignorance of the value of the attribute. The slot for attribute at_i is instantiated with this membership function when there is no information concerning the value of at_i for hypothesis h.

Two problem domains are considered throughout this report to illustrate the representations and the computations used in the GCT. The simple domain Γ introduced in Example 1 will be used in Section 3 to demonstrate the computations required for the generation of the support measures. Example 2 presents the more complicated

problem domain of representing radar types by their signal characteristics. This domain will be used in Section 4 to illustrate the features of the GCT.

Example 1: The domain knowledge base Γ consists of the four objects h_1, h_2, h_3, and h_4. The objects in Γ are defined by attributes at_1 and at_2 that assume values in the sets $V_1 = \{a, b, c, d, e\}$ and $V_2 = \{w, x, y, z\}$, respectively.

$$\begin{aligned}
\Gamma \quad h_1 : \widetilde{at_1}(h_1) &= 0/a + 0.5/b + 1/c + 0.5/d + 0/e \\
\widetilde{at_2}(h_1) &= 0/w + 0/x + 1/y + 1/z \\
\\
h_2 : \widetilde{at_1}(h_2) &= 1/a + 1/b + 0/c + 0/d + 0/e \\
\widetilde{at_2}(h_2) &= 0.5/w + 1/x + 0.5/y + 0/z \\
\\
h_3 : \widetilde{at_1}(h_3) &= 0/a + 0/b + 0.5/c + 1/d + 0.5/e \\
\widetilde{at_2}(h_3) &= 0.5/w + 0.5/x + 1/y + 0.5/z \\
\\
h_4 : \widetilde{at_1}(h_4) &= 0/a + 0/b + 1/c + 1/d + 0/e \\
\widetilde{at_2}(h_4) &= 0/w + 1/x + 1/y + 1/z
\end{aligned}$$

The partial membership function $\widetilde{at_1}(h_1)$ indicates imprecise knowledge of attribute 1 for object h_1. Values b, c, and d are all consistent with the domain information, but c is the most likely.

Example 2: Identifying a radar using information obtained through passive sensors employs signal characteristics to generate the measure of support for the alternatives. The characteristics used throughout examples in this report include radio frequency (rf), pulse width (pw), pulse repetition interval (pri), and scan type (sc). The signal characteristics of the emitters that comprise the emitter knowledge base have been provided in terms of linguistic terms such as *possible*, *probable*, and *most likely*. The high level description of emitter type 1 is shown below.

Emitter type 1

Radar Frequency (RF) Details

(a) The possible limits are 2.30 - 3.47 GHz.
The probable operating limits are 2.54 - 3.24 GHz.
The most observed band limits are 2.77 - 3.00 GHz.

(b) The RF is nominally constant.

Pulse Repetition Interval (PRI) Details

(a) The radar operates in either a constant PRI mode or a 2-element, 2-position stagger mode.

(b) In constant PRI mode, PRI limits are
   (i) 90.96 - 101.27 μsec
   (ii) 116.87 - 133.62 μsec
   (iii) 179.19 - 199.22 μsec

(c) In staggered mode, the PRI elements are
   (i) element 1: 66.42 - 71.11 μsec
   (ii) element 2: 136.36 - 157.57 μsec
   or
   (i) element 1: 74.76 - 107.90 μsec
   (ii) element 2: 191.08 - 194.09 μsec

Pulse Width (PW) Details

(a) The limits are 0.2 - 0.3 μsec.

Scan Details

(a) The scan is either conical (CON) or bidirectional sector (BDS).

(b) The conical scan period is between 0.04 and 0.06 sec.

(c) The bidirectional scan period lies between 2.0 and 4.0 sec.

Fuzzy representation provides the ability to translate such inherently imprecise linguistic descriptions into mathematical quantities. The frame-structure of an element of the radar knowledge base and the details of the construction of membership functions are given in Section 4.2.

2.2 Evidential Representation

The identification process of evidential reasoning systems utilizes information obtained from observations or sensors to generate a measure of support for each of the

hypotheses. The initial step in the processing of information is to transform the data returned by the sensor into a formalism that may be employed by the GCT. The conditional properties of possibility theory are utilized in developing an interpretation and representation of the observations that are processed by the classification system. The precision of an observation is determined by the capabilities and accuracy of the sensors that produce the information. Throughout this report we make an important distinction between *observation* and *evidence*. By observation we mean the uninterpreted information provided by a sensor. Evidence is the interpretation of the observation that is utilized by the reasoning system.

In the GCT, evidence concerning an attribute is represented as a possibility distribution over the set of possible values of the attribute. The evidential interpretation specifies a set of values that are consistent with the observation. An observation e that describes the properties of attribute at_i will be represented by a normal possibility distribution Π_e over V_i, the set of values of at_i. An evidential distribution Π_e is defined by a function π_e from V_i into [0,1]. The distribution Π_e provides the relationship between the observation and the values of the attribute. The value $\pi_e(v_j)$ indicates the degree of support, based on the observation e, that the value of attribute at_i is v_j.

Although they have the same mathematical form, it is important to recognize the subtle difference between the notion of partial membership function and possibility distribution. The value $\mu_{at_i(h)}(v_j)$ of a partial membership function indicates the certainty in the belief that the value of attribute at_i is v_j for object h. Conversely, the value $\pi_e(v_j)$ of the evidential possibility distribution Π_e represents the possibility of v_j given that information e is known.

A possibility distribution Π_e is an interpretation of the 'meaning' of an observation. This may be obtained from statistical data when available. Otherwise, it may be obtained from a subjective analysis of the problem domain and the reliability and precision of the sensor from which observation is obtained. Information is obtained by a mechanical sensor or a human observation that examines one characteristic (attribute) of the unknown object. The observation can be likened to a snapshot taken at a precise time with a single focal point. Imprecision is introduced by the capabilities and limitations of the sensor that provides the information.

3 Evidential Belief Measures

The preceding section reviewed the representations of domain and evidential information used in the GCT. The identification process is comprised of the acquisition of information and the generation of support for the elements in the domain knowledge base based on the acquired information. Support for the alternatives is obtained by comparing the evidential distribution Π_e concerning an attribute at with the partial membership descriptions $\widetilde{at}$ of the domain objects. The generation of a measure of support is a multi-step process whose computations depend on the technique employed to combine evidential support. This process can be summarized by the six steps discussed below:

- The acquisition of information e describing the characteristics of an attribute at_i of the unknown object.
- The construction of an evidential possibility distribution Π_e that interprets the raw data in the observation e.
- The construction of a measure of compatibility of the elements of the frame of discernment with the information in e. The result is a possibility distribution $\Pi(e)$ over Θ.
- The translation of the possibility distribution $\Pi(e)$ into the support measure required by the particular reasoning paradigm.
- The combination of evidence. If $e_1, \ldots, e_k$ are the observations that have been obtained, the individual measures of support $\Pi(e_1), \ldots, \Pi(e_k)$ must be combined to produce a measure of support based on the totality of the evidence.
- Interpreting the updated support measure to obtain a ranking of the elements in Θ.

The form of the measure of support constructed in step four is determined by the reasoning paradigm employed. For probabilistic systems, the measure of support is a probability distribution over frame of discernment. Similarly, in fuzzy reasoning systems the measure of support is a possibility distribution. The measure of support in a system using the Dempster-Shafer theory of evidential reasoning is a basic probability assignment over the subsets of the frame of discernment.

The techniques used to acquire and interpret observations in the GCT are presented in Section 4. In this section we review the computations required to update the measure of support and produce a ranking of the alternatives. The construction of $\Pi(e)$ from evidence Π_e is identical for fuzzy analysis and the Dempster-Shafer approach. The generation of $\Pi(e)$ from observation e is given in Section 3.1. Section

3.2 and 3.3 illustrate the remainder of the computation for fuzzy evidential reasoning and the Dempster-Shafer theory respectively. The probabilistic approach, which requires a transformation of the information in both the domain knowledge base and the evidential information, is described in Section 3.4.

3.1 Evidential Compatibility

The identification of an element of the frame of discernment is accomplished by the acquisition and analysis of evidence specifying the characteristics of the object. As described in the preceding sections, domain information concerning an attribute *at* is represented by a partial membership function

$$\mu_{at(h)} : V \to [0,1]$$

and evidence by a possibility distribution

$$\pi_e : V \to [0,1]$$

where V is the set of possible values of the attribute *at*. The compatibility of a hypothesis h with evidence e is obtained from the relationship of these two functions from V into $[0,1]$.

To motivate the properties of the generation of evidential support, it is useful to view domain information and evidence concerning an attribute *at* with values V as vectors of length $|V|$. The focal elements of the functions are the components of the vector with non-zero values. The compatibility of an object with evidence is determined by a component-wise analysis of the vectors. The focal set of the evidence e is the subset of V that is consistent with the acquired information. The only support for h given by Π_e occurs when both $\pi(v_i)$ and $\mu_{at(h)}(v_i)$ are non-zero, the intersection of the focal sets. The remainder of the support of e is attributed to values that are not consistent with the domain data.

Although we approach evidential reasoning via fuzzy set theory, the preceding criterion also holds for the generation of support using probabilistic representations. Probabilistic domain information for an attribute $\widetilde{at}(h)$ with possible values V is given by a probability distribution $p(h|v_1), \ldots, p(h|v_n)$. Similarly, evidence is a conditional distribution $p(v_1|e), \ldots, p(v_n|e)$. The compatibility of h with e, $p(h|e)$, is obtainable from these distributions. Support for h given e is given by

$$\begin{aligned} p(h|e) &= \sum_{i=1}^{n} p(h|v_i)p(v_i|e) \\ &= \sum_{v_j \in X} p(h|v_i)p(v_i|e) \end{aligned}$$

where $X \subseteq V$ is the set consisting of the elements for which both $p(h|v_i) > 0$ and $p(v_i|e) > 0$. That is, the probabilistic equivalent of the intersection of the focal sets.

A function that computes the compatibility of domain objects with evidence has two arguments: an attribute description and an evidential distribution. Zadeh [12] and Terano and Sugeno [10] have proposed *sup-min composition* as an estimate of the possibility of proposition given information.

$$\Pi(e) = Sup_{v \in V}[\mu_{at(h)}(v) \wedge \pi_e(v)]$$

Sup-min composition is an extension of fuzzy conjunction to distributions. The minimum operator performs an element-wise conjunction. Taking the supremum then chooses the maximal agreement between the domain information and the evidence.

The other compatibility function to be considered is the *expected value* operator. The expected value operator incorporates the influence of all the elements in the intersection of the focal sets of Π_e and $\widetilde{at}(h)$.

$$\Pi(e) = \frac{1}{sig(e)} \sum_{v \in V} (\mu_{at(h)}(v) \cdot \pi_e(v))$$

The product $\mu_{at}(v) \cdot \pi_e(v)$ measures the agreement of the domain information describing the likelihood of v with the support for v given by the evidence Π_e. The sum is then scaled by the sigma-count of the evidence $sig(e)$, the total of evidential support for all the possibilities. The result is the summation of terms of the form

$$\frac{\pi_e(v)}{sig(e)} \cdot \mu_{at(h)}(v)$$

for every $v \in V$. The quotient may be interpreted as the proportion of the possibility distribution of the evidence assigned to v.

The calculation of compatibility of evidence with information is illustrated in Example 4. A comparison of several functions for determining the compatibility of domain objects with evidential information can be found in [7]. This work shows that the expected value operator constructs distributions that are superior discriminators than those produced by sup-min composition.

Example 4: Domain knowledge base Γ from Example 1 and evidential distributions Π_e containing information describing attribute 1 and Π_f containing information describing attribute 2 are used to demonstrate the construction of compatibility distributions $\Pi(e)$ and $\Pi(f)$.

$$\Pi_e = 0.5/a + 1/b + 0.5/c + 0/d + 0/e$$
$$\Pi_f = 1/w + 1/x + 0/y + 0/z$$

evidence e	h_1	h_2	h_3	h_4
sup-min composition	0.5	1	0.5	0.5
expected value	0.5	0.75	0.125	0.25

evidence f	h_1	h_2	h_3	h_4
sup-min composition	0	1	0.5	1
expected value	0	0.75	0.5	0.5

The effects of the incorporation of the entire focal set of the evidence can be seen by examining the distributions $\Pi(e)$ obtained from Γ and Π_e. Using sup-min composition, h_3 is assigned compatibility 0.5 since $\mu_{at_1(h_3)}(c) = 0.5$ and $\pi_e(c) = 0.5$. The compatibility assigned by the expected value operator is much lower since the majority of the support indicated by e is assigned to a and b, which are incompatible with h_3.

3.2 Fuzzy Support Updating

Updating support using fuzzy reasoning techniques is a natural extension of the evidential compatibility calculations. In a classification system based on fuzzy reasoning, the measure of support is a possibility distribution $\Pi(E)$ over Θ that indicates the possibility of the objects with the accumulated evidence $E = \{e_1, \ldots, e_k\}$. The composite distribution $\Pi(E)$ is obtained by combining possibility distributions of the form $\Pi(e_i)$ generated by the observations $e_i, 1 \leq i \leq k$.

Possibility distributions $\Pi(e)$ and $\Pi(f)$ over Θ specify the support for the objects in Θ constructed from evidence Π_e and Π_f respectively. The combination of support in fuzzy reasoning utilizes the minimum function, the fuzzy analog of logical conjunction. Thus the possibility distribution $\Pi(e, f)$ that specifies the combined support based on observations e and f is given by

$$\pi_{(e,f)}(h) = \pi_e(h) \wedge \pi_f(h),$$

Support updating using the minimum operator is commutative and associative, ensuring a unique possibility distribution for each combination of evidence regardless of the order in which the information is obtained and processed. The objects are ranked according to the support indicated by the possibility distribution $\Pi(E)$ that is obtained by combining all the acquired information.

Example 5: The combined support for the objects in Γ are obtained using evidence Π_e and Π_f and the compatibility measures $\Pi(e)$ and $\Pi(f)$ derived in Example 4.

$\Pi(e, f)$	h_1	h_2	h_3	h_4
sup-min composition	0	1	0.5	0.5
expected value	0	0.75	0.125	0.25

Regardless of the compatibility function used to generate $\Pi(e)$ and $\Pi(f)$, h_2 is deemed the object most compatible with evidence e and f. Since the updated possibility function need not be a normal possibility distribution, it is the relative strength of the support rather than the actual numeric value that should be used as a measure of support. With this in mind, we see that the results obtained using the compatibility distributions constructed by the expected value operator supports h_2 to a greater degree than the support assigned using sup-min composition to determine evidential compatibility.

3.3 Dempster-Shafer Updating

The Dempster-Shafer (D-S) theory of evidential reasoning [6] employs a set-based measure of support and Dempster's rule for updating. The representation of evidential support in the D-S approach is a *basic probability function.* A basic probability assignment is a function $m : 2^{\Theta} \rightarrow [0, 1]$ that satisfies

i) $m(\emptyset) = 0$

ii) $\sum_{A \in 2^{\Theta}} m(A) = 1.$

The value $m(A)$ assigned to a subset A of the frame of discernment Θ indicates the support for the elements of A. That is, $m(A)$ is the measure of the support, based on the evidence represented by m, that the object being identified is an element of A. This portion of the total support cannot be further subdivided among the subsets of A. Support expressly for the elements of a set $B \subset A$ is given by $m(B)$. The set based support measure has been proposed because of the increased representational capabilities over those of point based measures.

The first step in the D-S evaluation is the transformation of the compatibility distribution $\Pi(e)$ generated from the evidential distribution Π_e into a basic probability assignment m_e. Recall that the distribution $\Pi(e)$ specifies the compatibility of the objects with the evidential distribution Π_e. Let $\{\alpha_1, \ldots, \alpha_k\}$ be the range of the compatibility distribution $\Pi(e)$. Without loss of generality, assume that $\alpha_i \geq \alpha_{i+1}$ for $i = 1, \ldots, k-1$. The α-support sets are used to define the focal sets of the basic probability assignment generated from $\Pi(e)$ and the domain information encapsulated in Θ. Definition 2 defines the basic probability assignment over Θ generated by evidence $\Pi(e)$.

Definition 1: The α-support of a compatibility distribution $\Pi(e)$ is the set

$$S(e, \alpha) = \{h_i \in \Theta \mid \pi_{(e)}(h_i) = \alpha\}.$$

Definition 2: Let $\Pi(e)$ be a compatibility distribution with range $\alpha_1, \ldots, \alpha_k$. The focal elements of the basic probability assignment constructed from $\Pi(e)$ are the sets $A_1, \ldots, A_k$ defined by

$$A_i = \bigcup_{j=1}^{i} S(e, \alpha_j).$$

The basic probability assignment m_e is obtained by solving the equations

i) $\sum_{i=1}^{k} m_e(A_i) = 1$

ii) $\dfrac{m_e(A_i)}{|A_i|} = \dfrac{\alpha_i}{\alpha_1} \dfrac{m_e(A_1)}{|A_1|}$.

where $|A|$ denotes the cardinality of the set A.

The focal elements of the basic probability assignment form a nested sequence $A_1 \subseteq A_2 \subseteq \cdots \subseteq A_k$. The equations in part ii) ensure that the support assigned to the focal elements is proportional to that assigned by the compatibility distribution $\Pi(e)$. Equation i) normalizes the distribution of support producing a basic probability assignment. The support assigned to the focal set A_i depends upon the number of elements in the set and the degree of support α_i. Dependence on cardinality of the focal sets differentiates this technique from the standard transformation of a possibility distribution to a basic probability assignment (see [5]).

Support combination rule of D-S theory utilizes the basic probability assignment representation of support. Let m_1 and m_2 be basic probability assignments with focal elements $A_1, \ldots, A_n$ and $B_1, \ldots, B_m$ respectively. Focal elements A_i and B_j are *compatible* if $A_i \cap B_j$ is nonempty. The intersection of A_i and B_j contains precisely the elements that are consistent with both A_i and B_j. Support for the set of consistent possibilities is obtained directly from the values $m_1(A_i)$ and $m_2(B_j)$. Utilizing an independence assumption, the combination rule assigns the product $m_1(A_i)m_2(B_j)$ to the intersection of the two focal elements.

Focal elements A_i and B_j are *incompatible* if their intersection is the empty set. The value

$$K = \sum_{A_i \cap B_j = \emptyset} m_1(A_i)m_2(B_j) \qquad (1)$$

measures the incompatibility of the basic probability assignments m_1 and m_2. The incompatibility measure K can assume values between zero and one. If $K = 1$, there are no compatible focal elements and the basic probability assignments are said to be *inconsistent*.

Definition 3: Let m_1 and m_2 be two basic probability assignments over a frame of discernment Θ. A new basic probability assignment m, called the *orthogonal sum* of m_1 and m_2, denoted $m_1 \oplus m_2$, is obtained by the computation

i) $m(\emptyset) = 0$

ii) $m(A) = \sum_{A_i \cup B_j = A} (m_1(A_i)m_2(B_j))/(1-K)$

for every nonempty subset A of Θ, where K is the measure of incompatibility of the constituent basic probability assignments.

The orthogonal sum calculation is generally known as Dempster's rule of combination. A nonempty set $A \subseteq \Theta$ is a focal element of the orthogonal sum if it is the intersection of a focal element of m_1 and a focal element of m_2. The pairwise intersection calculation may assign a nonzero value to the empty set. Setting $m(\emptyset)$ to 0 and scaling by $1-K$ produces a basic probability assignment.

Updating support in a D-S evidential reasoning system uses Dempster's rule to combine basic probability assignments. If m_E is the basic probability assignment that indicates the support obtained from the observations $E = \{e_1, \ldots, e_n\}$ and m_e the basic probability assignment representing the support for additional information e, the combined support $m_{(E,e)}$ is $m_E \oplus m_e$.

A basic probability assignment specifies the support for the elements of a set that cannot be distributed to proper subsets of the set. The final step in the generation of evidential support is to rank the alternatives based on the evidence. D-S theory provides two measures, *belief* and *plausibility*, to rank the objects based on the support in a basic probability assignment m_E.

A *belief function* is a function $bel : 2^\Theta \rightarrow [0,1]$ whose values indicate the total support for the elements of a set. A belief function can be obtained from the values of a basic probability assignment m_E. For every $A \subseteq \Theta$, the value $bel(A)$ is defined by

$$bel(A) = \sum_{B \subseteq A} m_E(B). \tag{2}$$

$bel(A)$ is the sum of the values assigned to the subsets of A. The summation accumulates all the support that is attributed to the elements of A. The plausibility expresses the measure of evidential support that is not against the elements of A. Using this terminology, the plausibility of a set A, denoted $pl(A)$, is the total of the nondisconfirming support.

$$pl(A) = 1 - bel(\overline{A})$$

It is important to recognize the difference between $bel(A)$ and $pl(A)$. $bel(A)$ is the total support for the elements of A. The plausibility is the total amount of belief in A that the evidence will permit. The difference between $pl(A)$ and $bel(A)$ can be considered the amount of uncertainty in the estimate of the likelihood of A.

The support assigned to the singleton set $\{h_i\}$ can be used as a measure of support for the object h_i. The effectiveness of using belief and plausibility as a ranking

function has been examined in [3]. Plausibility has been shown to be more effective and has been utilized in the GCT. The D-S analysis of evidence Π_e and Π_f in problem domain Γ is given in Example 6 (see examples 1 and 4).

Example 6: The construction of α-support sets with imprecise domain information is demonstrated using the domain Γ and compatibility distribution $\Pi(e) = 0.5/h_1 + 0.75/h_2 + 0.125/h_3 + 0.25/h_4$ produced using the expected value operator (Example 5). The focal sets of the corresponding basic probability assignment are

$$\begin{aligned}
s(e, 0.75) &= A_1 = \{h_2\} \\
s(e, 0.5) &= A_2 = \{h_1, h_2\} \\
s(e, 0.25) &= A_3 = \{h_1, h_2, h_4\} \\
s(e, 0.125) &= A_4 = \{h_1, h_2, h_3, h_4\}.
\end{aligned}$$

Definition 2 produces the set of linear equations

$$\begin{aligned}
&m_e(A_1) + m_e(A_2) + m_e(A_3) + m_e(A_4) = 1 \\
&m_e(A_2) = 2(0.5/0.75)m_e(A_1) \\
&m_e(A_3) = 3(0.25/0.75)m_e(A_1) \\
&m_e(A_4) = 4(0.125/0.75)m_e(A_1).
\end{aligned}$$

Solving the system of equations produces

$$\begin{aligned}
m_e(A_1) &= 0.25 \\
m_e(A_2) &= 0.33 \\
m_e(A_3) &= 0.25 \\
m_e(A_4) &= 0.17.
\end{aligned}$$

The compatibility distribution $\Pi(f) = 0/h_1 + 0.75/h_2 + 0.5/h_3 + 0.5/h_4$ generates the basic probability distribution

$$\begin{aligned}
m_f(\{h_2\}) &= 0.33 \\
m_f(\{h_2, h_3, h_4\}) &= 0.66.
\end{aligned}$$

The basic probability assignment that designates the support for the elements of Θ based on evidence e and f is obtained by combining m_e and m_f using Dempster's rule.

$$\begin{aligned}
m_{(e,f)}(\{h_2\}) &= \tfrac{13}{18} \\
m_{(e,f)}(\{h_2, h_4\}) &= \tfrac{3}{18} \\
m_{(e,f)}(\{h_2, h_3, h_4\}) &= \tfrac{2}{18}.
\end{aligned}$$

Ranking the candidates uses the plausibility assigned to the singleton sets by the basic probability function $m_{(e,f)}$.

object	h_1	h_2	h_3	h_4
plausibility	0	1	0.11	0.17

As with the fuzzy analysis of Γ with observations e and f, h_2 receives the most support. The independence assumption of Dempster's rule reduces the support for elements that are partially supported by the constituent basic probability assignment.

3.4 Probabilistic Evaluation

The generation of support using probabilistic techniques requires a transformation of the domain and evidential representations into probability distributions. The fuzzy domain knowledge base is converted into a set of conditional probability distributions. Evidence is also transformed into a probability distribution. The objective is to compute a probability distribution $P(\Theta = h_i \mid E)$ where E is the accumulation of evidence that has been acquired and processed. The algorithm to determine support is based on probabilistic expectation. The updating may be viewed as a series of matrix operations or as a Bayesian network. In this section we will present the matrix development of probabilistic evidential support updating. The interpretation of this algorithm as a Bayesian network can be found in [1].

The computations involved in determining probabilistic support will be illustrated using the domain knowledge base Γ of Example 1 and the evidence for the objects in Γ given in Example 4.

The objects in Γ are defined by two attributes at_1 and at_2 with domains $\{a, b, c, d, e\}$ and $\{w, x, y, z\}$ respectively. Domain knowledge about the characteristics of the objects is represented as conditional probability distributions over the possible attribute values. Each attribute defines one matrix. The conditional probability distribution for attribute at_1 given Θ, written $P(at_1 \mid \Theta)$, as a matrix is

$$\begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array}
\begin{array}{c} \begin{array}{cccc} h_1 & h_2 & h_3 & h_4 \end{array} \\
\left[\begin{array}{cccc} 0 & 0.5 & 0 & 0 \\ 0.25 & 0.5 & 0 & 0 \\ 0.5 & 0 & 0.25 & 0.5 \\ 0.25 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0.25 & 0 \end{array}\right] \\
[P(at_1 \mid \Theta)] \end{array}$$

The column designated by h_i in the domain knowledge matrix for attribute at_1 gives the distribution that defines the probability of the attribute assuming a certain value given that the object is known to be h_i. That is, $P(at_i \mid \Theta)$ is represented by the matrix $[p(a_r \mid h_i)]$ where r indexes rows and i columns.

The knowledge base for at_2, written $P(at_2 \mid \Theta)$, reformulated as a matrix is

$$\begin{array}{c} \\ w \\ x \\ y \\ z \end{array}
\begin{array}{c} \begin{array}{cccc} h_1 & h_2 & h_3 & h_4 \end{array} \\
\left[\begin{array}{cccc} 0 & 0.25 & 0.2 & 0 \\ 0 & 0.5 & 0.2 & 0.333 \\ 0.5 & 0.25 & 0.4 & 0.333 \\ 0.5 & 0 & 0.2 & 0.333 \end{array}\right] \\
[P(at_2 \mid \Theta)] \end{array}$$

Once the domain information has been converted to its probabilistic representation, the evidence to be analyzed must also be transformed into a probabilistic

representation. Evidence concerning an attribute at_i is represented as a conditional probability distribution over the set V_i of possible values of at_i. Let E be a random variable over the domain of the attribute. The evidence about at_1 is obtained by normalizing the possibility distribution Π_e from Example 4.

$$P(at_1 \mid E{=}e) \;=\; \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{bmatrix} 0.25 & 0.5 & 0.25 & 0 & 0 \end{bmatrix} \end{array}. \tag{3}$$

Similarly evidence Π_f about at_2 produces the distribution

$$P(at_2 \mid F{=}f) \;=\; \begin{array}{c} \begin{array}{cccc} w & x & y & z \end{array} \\ \begin{bmatrix} 0.5 & 0.5 & 0 & 0 \end{bmatrix} \end{array} \tag{4}$$

The probabilistic generation of support begins by assigning initial probability values to the objects in Θ. This may be done using *a priori* information. If no such information exists, then each object is considered equally likely. Using the latter approach, the probability distribution $P(\Theta)$ initialized and represented by the 1-by-4 matrix *i.e.,* a vector.

$$P(\Theta) \;=\; \begin{array}{c} \begin{array}{cccc} h_1 & h_2 & h_3 & h_4 \end{array} \\ \begin{bmatrix} 0.25 & 0.25 & 0.25 & 0.25 \end{bmatrix} \end{array}$$

The evidence generated by observations e and f and represented by the vectors $P(at_1 \mid E{=}e)$ and $P(at_2 \mid F{=}f)$ is incorporated into the measure of support sequentially. The computation begins by finding $P(\Theta \mid at_1)$, the support for the objects given values of attribute at_1. We use Bayes' rule to form $P(\Theta \mid at_1)$ from $P(\Theta)$ and $P(at_1 \mid \Theta)$.

$$P(\Theta \mid at_1) \;=\; \frac{P(\Theta)P(at_1 \mid \Theta)}{\sum_\Theta P(\Theta)P(at_1 \mid \Theta)} \tag{5}$$

or, dropping the normalization,

$$P(\Theta \mid at_1) \;\propto\; P(\Theta)P(at_1 \mid \Theta) \tag{6}$$

Written element by element, this is

$$p(h_i \mid a_k) \;\propto\; p(h_i) \cdot p(a_k \mid h_i) \tag{7}$$

over all i for fixed k. Letting $\otimes$ represent the matrix operation corresponding to Equation 7, have

$$\underbrace{\begin{array}{c} \begin{array}{cccc} h_1 & h_2 & h_3 & h_4 \end{array} \\ \begin{bmatrix} 0.25 & 0.25 & 0.25 & 0.25 \end{bmatrix} \end{array}}_{[P(\Theta)]} \;\otimes\; \underbrace{\begin{array}{c|c} & \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{array}{c} h_1 \\ h_2 \\ h_3 \\ h_4 \end{array} & \begin{bmatrix} 0 & 0.25 & 0.5 & 0.25 & 0 \\ 0.5 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0.25 & 0.5 & 0.25 \\ 0 & 0 & 0.5 & 0.5 & 0 \end{bmatrix} \end{array}}_{[P(at_i \mid \Theta)]^T}$$

$$
= \begin{array}{c} \\ h_1 \\ h_2 \\ h_3 \\ h_4 \\ \\ \end{array}
\begin{array}{ccccc} a & b & c & d & e \\ \left[\begin{array}{ccccc} 0 & 0.063 & 0.125 & 0.063 & 0 \\ 0.125 & 0.125 & 0 & 0 & 0 \\ 0 & 0 & 0.063 & 0.125 & 0.063 \\ 0 & 0 & 0.125 & 0.125 & 0 \end{array}\right] \\ \hline 0.125 & 0.188 & 0.313 & 0.25 & 0.063 \end{array}
$$

$$
\left(\left[P(\Theta)\right] \otimes \left[P(at_1 \mid \Theta)\right]\right)^T
$$

the columns of which are proportional to $\left[P(\Theta \mid at_1)\right]$. (We maintain the conventional row-column conformance of operands. Accordingly the transposed matrix, $\left[P(\Theta \mid at_1)\right]^T$, is shown in the computation.) Normalizing, dividing each element in the column by the column sum shown above, produces $P(\Theta \mid at_1)$.

$$
P(\Theta \mid at_1) \quad = \quad \begin{array}{c|ccccc} & a & b & c & d & e \\ \hline h_1 & 0 & 0.333 & 0.4 & 0.25 & 0 \\ h_2 & 1 & 0.667 & 0 & 0 & 0 \\ h_3 & 0 & 0 & 0.2 & 0.5 & 1 \\ h_4 & 0 & 0 & 0.4 & 0.5 & 0 \end{array} \tag{8}
$$

Because we have chosen to begin the computation with a uniform distribution over Θ, the distribution $P(\Theta \mid at_1)$ could have been computed by normalizing $P(at_1 \mid \Theta)$ directly. However, this is not true in general and does not hold in general for further steps in the iteration.

The probability matrix $P(\Theta \mid at_1)$ is used to compute $P(\Theta \mid E{=}e)$, the updated support for the objects in Θ, using the product rule

$$
P(\Theta \mid E{=}e_1) \quad = \quad \sum_A P(at_1 \mid E{=}e_1) P(\Theta \mid at_1). \tag{9}
$$

The corresponding operation is matrix multiplication, *i.e.*,

$$
\underset{\left[P(at_1 \mid E=e_1)\right]}{\begin{array}{ccccc} a & b & c & d & e \\ \left[0.25\right. & 0.5 & 0.25 & 0 & \left.0\right] \end{array}}
\quad \cdot \quad
\underset{\left[P(\Theta \mid at_1)\right]^T}{\begin{array}{c|cccc} & h_1 & h_2 & h_3 & h_4 \\ \hline a & 0 & 1 & 0 & 0 \\ b & 0.333 & 0.667 & 0 & 0 \\ c & 0.4 & 0 & 0.2 & 0.4 \\ d & 0.25 & 0 & 0.5 & 0.5 \\ e & 0 & 0 & 1 & 0 \end{array}}
$$

$$
= \quad \underset{\left[P(\Theta \mid E=e_1)\right]}{\begin{array}{cccc} h_1 & h_2 & h_3 & h_4 \\ \left[0.267\right. & 0.583 & 0.050 & \left.0.100\right] \end{array}}
$$

Hence $P(\Theta \mid E{=}e)$ is taken as the updated support for hypotheses.

Upon the acquisition of additional information f, the support must be updated to represented the accumulated information. The computation to update $P(\Theta \mid E{=}e)$ given evidence f proceeds using the same steps as the previous computation. In this case, the initial distribution is the current state of belief $P(\Theta \mid E{=}e)$. The resulting updated support is $P(\Theta \mid E{=}e, F{=}f)$. First the probability of the attribute values of at_2 are incorporated into the support to form $P(\Theta \mid at_2, E{=}e)$.

$$\underset{[P(\Theta \mid E=e)]}{\begin{array}{cccc} h_1 & h_2 & h_3 & h_4 \\ \end{array} \atop [0.267 \;\; 0.583 \;\; 0.050 \;\; 0.100]} \otimes \underset{[P(at_2 \mid \Theta)]^T}{\begin{array}{c|cccc} & w & x & y & z \\ \hline h_1 & 0 & 0 & 0.5 & 0.5 \\ h_2 & 0.25 & 0.5 & 0.25 & 0 \\ h_3 & 0.2 & 0.2 & 0.4 & 0.2 \\ h_4 & 0 & 0.333 & 0.333 & 0.333 \end{array}}$$

$$= \begin{array}{c|cccc} & w & x & y & z \\ \hline h_1 & 0 & 0 & 0.134 & 0.134 \\ h_2 & 0.146 & 0.292 & 0.146 & 0 \\ h_3 & 0.01 & 0.01 & 0.02 & 0.01 \\ h_4 & 0 & 0.033 & 0.033 & 0.033 \\ \hline & 0.156 & 0.335 & 0.333 & 0.177 \end{array}$$

$$\propto [P(\Theta \mid at_2, E{=}e)]$$

$$\xrightarrow{\text{normalizing}} \begin{array}{c|cccc} & w & x & y & z \\ \hline h_1 & 0 & 0 & 0.045 & 0.757 \\ h_2 & 0.936 & 0.872 & 0.049 & 0 \\ h_3 & 0.064 & 0.03 & 0.007 & 0.056 \\ h_4 & 0 & 0.099 & 0.01 & 0.188 \end{array}$$

$$[P(\Theta \mid at_2, E{=}e)]$$

In the second part of the computation, $P(\Theta \mid at_2, E{=}e)$ and the evidence in Equation 4 are combined to form $P(\Theta \mid E{=}e, F{=}f)$.

$$P(\Theta \mid E{=}e, F{=}f) = \sum_{W} P(at_2 \mid F{=}f) P(\Theta \mid at_2, E{=}e)$$

or

$$\overset{\begin{array}{cccc} w & x & y & z \end{array}}{[0.5 \;\; 0.5 \;\; 0 \;\; 0]} \cdot [P(\Theta \mid at_2, E{=}e)]^T = \overset{\begin{array}{cccc} h_1 & h_2 & h_3 & h_4 \end{array}}{[0 \;\; 0.904 \;\; 0.047 \;\; 0.05]}$$

The resulting matrix represents the support for the objects based on the combination of the evidence. The probabilistic generation of support consists of repeated iterations of the preceding cycle of algebraic manipulations of probability distributions.

4 Generic Classification Tool

The Generic Classification Tool (GCT) has been designed to provide an environment for analyzing the effectiveness and efficiency of evidential reasoning paradigms. The GCT consists of three major subsystems: the Database System, the Observation-Interpretation System, and the Reasoning System. The Database System is used to create domain knowledge bases, which contain the formal representation of the domain specific information. This information is represented using the fuzzy techniques described in Section 2.1. The Observation Interpretation subsystem is used to create an observation-interpretation (OI) knowledge base that describes the properties of the information that is obtained by the sensors. The OI knowledge base is used to transform the observed data into a possibility distribution. The resulting possibility distribution is then used by the various reasoning paradigms. The Reasoning System is the heart of the evidential reasoning process. Evidence, generated by an observation and the information in the OI knowledge base, is analyzed using the selected support generation paradigm to produce a measure of belief for the candidates in the frame of discernment.

As illustrated in Figure 1, the Reasoning System consists of three main subsystems: the Controller, the Evidential Interpreter and the Support Generation System. The Controller obtains the observation and provides it to the Evidential Interpreter. Using the OI knowledge base, the Evidential Interpreter transforms the observation into evidence.

The evidence is passed to the Support Generation System, which computes the compatibility of the evidence with elements in the domain knowledge base. The compatibility measure is then transformed into the measure of support of the particular evidential reasoning paradigm. The new evidence is combined with current state of belief to form an updated support value for each object in the domain knowledge base. The Controller transmits the updated support measures to a result file which may then be processed by a set of analysis programs. The following sections describe each of the major components of the GCT.

4.1 Major Subsystems

4.1.1 Database System

The Database System is used to define the properties of the elements of the frame of discernment and to create a domain knowledge base. The process begins by defining the frame structures of the objects to be recorded in the knowledge base. The following information must be specified to produce the frame structure that provides the skeleton for the objects that comprise the domain knowledge base.

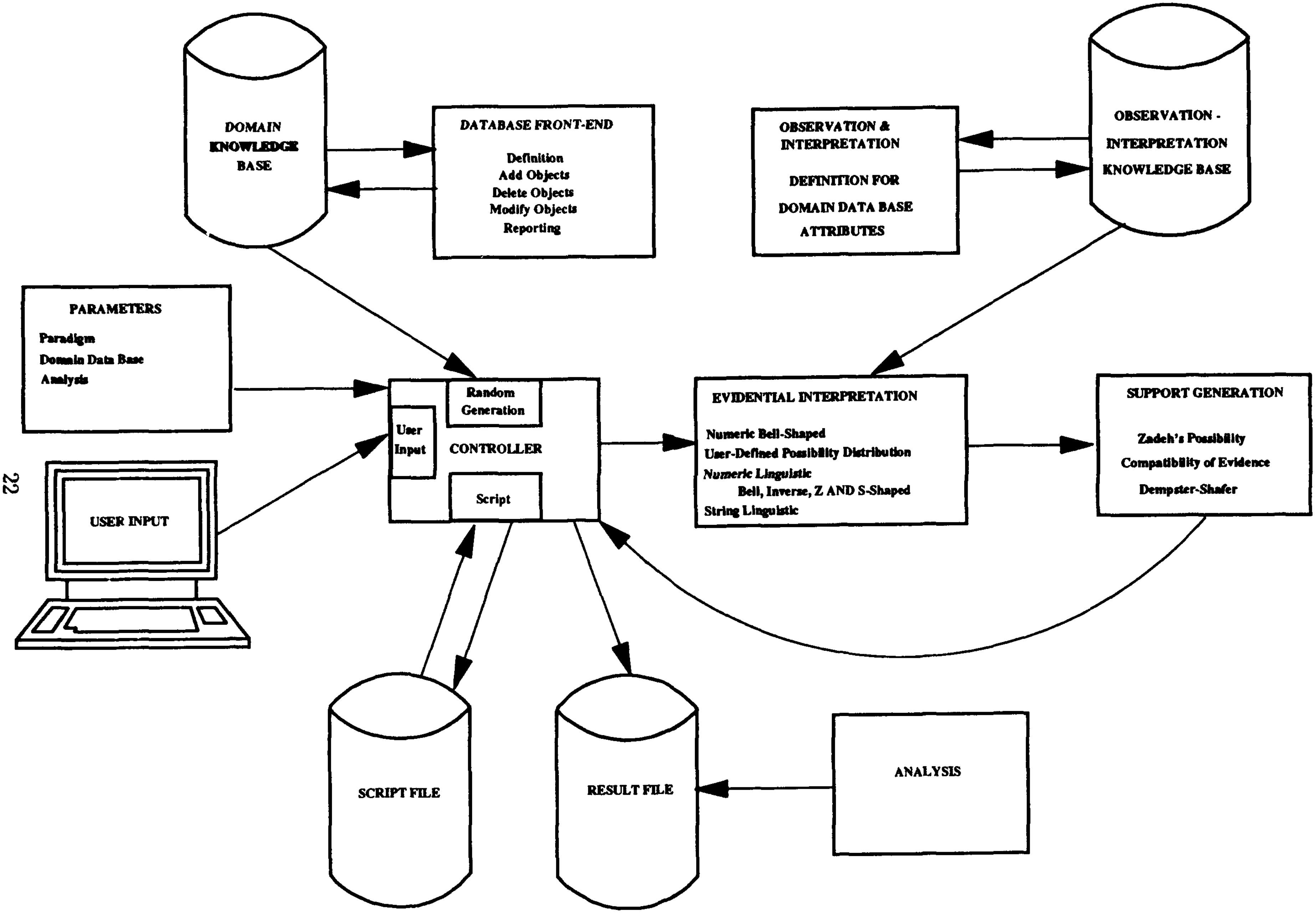

Figure 1: Generic Classification Tool architecture

class The class of objects represented in the knowledge base.

attributes The characteristics that describe and differentiate the objects. These define the structure of the knowledge base.

values The type and range of the values that each attribute may assume.

Attributes are initially assigned a name and a data type. The data types are INTEGER, FLOATING POINT, and STRING. This is followed by specifying the values that each attribute may assume. The set of permissible values for a string attribute must be provided explicitly. For numeric attributes, the range may be defined by upper and lower bounds. The following data will be requested when defining the domain knowledge base.

name:	the name for the attribute
data type:	INTEGER, FLOATING POINT or STRING (enumeration)
low value:	for INTEGER and FLOATING POINT
high value:	for INTEGER and FLOATING POINT
legal values:	for STRING
ordered:	for STRING
multi-valued:	for STRING
range-step:	for INTEGER and FLOATING POINT

An attribute is *multi-valued* if its possible values are not considered to be mutually exclusive. The *range-step* defines the precision to which the domain information must be provided in numeric data types. A range step of i for an integer data type with lower bound n and upper bound m indicates that information is required for attribute values n, $n+i$, $n+2i$, $\ldots$, m.

A string attribute may be ordered or unordered. An attribute is ordered if there is a linear proximity relation on the values. These relations are used in interpreting and processing imprecise information. Example 7 gives the definitions of the attributes for the "radar" knowledge base that will be used throughout the sequel.

Example 7: A knowledge base containing the physical characteristic of six emitter types has been constructed to illustrate the properties of the GCT. The full knowledge base is described in Appendix B. The frame for the domain objects is defined as follows:

Attribute	Data Type	Range
RF	FLOATING POINT	2.08 to 3.97 GHz
PW	FLOATING POINT	0.15 to 1.0 usec
PRI	FLOATING POINT	53.25 to 223.32 usec
PRI-ELEMENT1	FLOATING POINT	53.25 to 122.80 usec
PRI-ELEMENT2	FLOATING POINT	63.70 to 194.09 usec
SCAN-PERIOD	FLOATING POINT	0.04 to 10.0 sec

Each of the above attributes has a range-step of 0.01.

Once the frame structure of the domain objects has been provided, objects may be created and added to the domain knowledge base. For each object and attribute, the user is asked to input the partial membership function that describes the properties of the object. Precise domain information may be entered by specifying a single value from the range of the attribute. Precise information is stored in the domain knowledge base as a partial membership function which assigns 1.0 to the specified value and all other values 0.

Imprecise domain information is entered by one of two methods: by explicitly entering a partial membership function or by using a predefined function generator that produces a partial membership function. Explicitly defined membership functions are entered as a list of pairs with the first element in the pair being an attribute value and the second the associated membership value. For example, the domain information for the scan period for emitter type 4 is represented by the list

```
((0.05 1.0) (0.06 1.0))
```

All items not specifically defined in the list are assigned membership value 0. That is, those values are completely incompatible with the properties of the object being defined.

A number of function generators are provided to produce common partial membership functions. The GCT predefined function generators are:

BELL-SHAPED:	INTEGER, FLOATING POINT
INVERSE-BELL-SHAPED:	for INTEGER and FLOATING POINT
S-SHAPED:	for INTEGER and FLOATING POINT
Z-SHAPED:	for INTEGER and FLOATING POINT
RANGE:	for ordered types
INEXACT:	for ordered types

The function generator BELL-SHAPED takes three arguments: the midpoint value of the peak, p; the peak range, α, and the compatibility range, β. The peak range is the distance from the midpoint to the end of the flat peak. The midpoint value of the peak is always assigned the compatibility 1.0 and all values in the peak range on either side of the midpoint have a possibility of 1.0. The compatibility range is the distance from the midpoint to the nearest value whose compatibility is 0. All values outside the compatibility range are assigned 0. A value within the compatibility range but not in the peak range has a value determined by the parameterized membership function which creates a curve joining the endpoints of the compatibility range and the peak range. Given a value p for an attribute, the fuzzy set representing the uncertain value of the attribute is defined by the parameterized membership function

$$\mu_{ATTR-1}(u) = \begin{cases} 0 & \text{if } u \leq p - \beta \\ [2/(\beta-\alpha)^2](u-p+\beta)^2 & \text{if } p - \beta \leq u \leq p - (\alpha+\beta)/2 \\ 1 - [2/(\beta-\alpha)^2](u-p+\alpha)^2 & \text{if } p - (\alpha+\beta)/2 \leq u \leq p - \alpha \\ 1 & \text{if } p - \alpha \leq u \leq p + \alpha \\ 1 - [2/(\beta-\alpha)^2](u-p-\alpha)^2 & \text{if } p + \alpha \leq u \leq p + (\alpha+\beta)/2 \\ [2/(\beta-\alpha)^2](u-p-\beta)^2 & \text{if } p + (\alpha+\beta)/2 \leq u \leq p + \beta \\ 0 & \text{if } u \geq p + \beta \end{cases}$$

where u takes on the values in the range of the attribute. The curve is a flat-topped bell curve which assigns a membership value of 1 to all $u \in [p-\alpha, \ldots, p+\alpha]$ and symmetrically decreases membership in the intervals $[p-\beta, \ldots, p-\alpha]$ and $[p+\alpha, \ldots, p+\beta]$ before reaching a minimum value of 0 for the intervals $[-\infty, \ldots, p-\beta]$ and $[p+\beta, \ldots, \infty]$. The parameter α represents the distance from the generated value p to the edge of the curve's peak. The parameter β specifies the distance from p at which the value of the membership function becomes 0.

For example, the value for the attribute RF for emitter type 4 might be specified as:

```
(BELL-SHAPED 2.27 4 20)
```

This defines a partial membership function in which all values between 2.23 and 2.31 have membership value 1.0. All values less than or equal to 2.07 and greater than or equal to 2.47 are assigned compatibility values of 0. The partial membership function is completed by connecting (2.07, 0) to (2.23, 1) and (2.31, 1) to (2.47, 0) using the defined bell-shaped function. The function obtained from INVERSE-BELL-SHAPED is the fuzzy complement of the function obtained from BELL-SHAPED.

The function generator S-SHAPED has two arguments: the smallest value whose assigned 1.0 and the distance from this value to the largest value whose compatibility value is 0. All attribute values greater than the leftmost value have a compatibility of 1.0 and all attribute values less than this largest value have compatibility 0. Attribute

values in the intermediate range are assigned compatibility values determined by connecting the endpoints of the two ranges using the S-SHAPED function definition. The Z-SHAPED function generator produces membership functions that are similar to those constructed by S-SHAPED with the intervals of total compatibility and incompatibility reversed.

The function generator INEXACT constructs partial membership functions for any ordered attribute. The single argument specifies one value which is assigned compatibility 1.0. The nearest values of the attribute, according to the linear ordering, are assigned 0.5. All other values are incompatible.

Another helpful function generator applicable for an attribute with ordered values is RANGE. The arguments to RANGE are a lower and an upper bound. RANGE defines a half-open interval with the upper bound excluded. All the values in the interval are assigned the same compatibility value. For example, the domain information for pulse width may be represented by the list

```
((RANGE 0.2 0.3) 1.0)
```

produces the partial membership function

$$1/0.20 + 1/0.21 + 1/0.22 + 1/0.23 + 1/0.24 + \ldots. + 1/0.28 + 1/0.29$$

since pulse width is defined with a range-step of 0.01.

The LISP language permits the addition of new function generators. Being able to define and redefine function generators provides the ability to easily expand and modify the information in the knowledge bases.

The Database System also provides the knowledge base management functions which allow the user to open, close, save, rename a knowledge base and to describe the contents of a knowledge base. A description of a domain knowledge base contains the frame structure and an enumeration of the objects currently in the knowledge base. In addition the Database System provides a fuzzy query capability that permits the user to perform a series of queries that use conjunction between the queries.

4.1.2 Observation-Interpretation System

The Observation-Interpretation subsystem is used to define the information required to transform an observation into the format to be processed by the reasoning system. The required format for an observation concerning an attribute is a possibility distribution over the set of values that the attribute may assume. The OI knowledge base is constructed to provide the means to interpret observations during the identification process.

An interpretation definition for an attribute may be specified using a GCT-defined or user-defined function generator or by explicitly specifying a possibility distribution

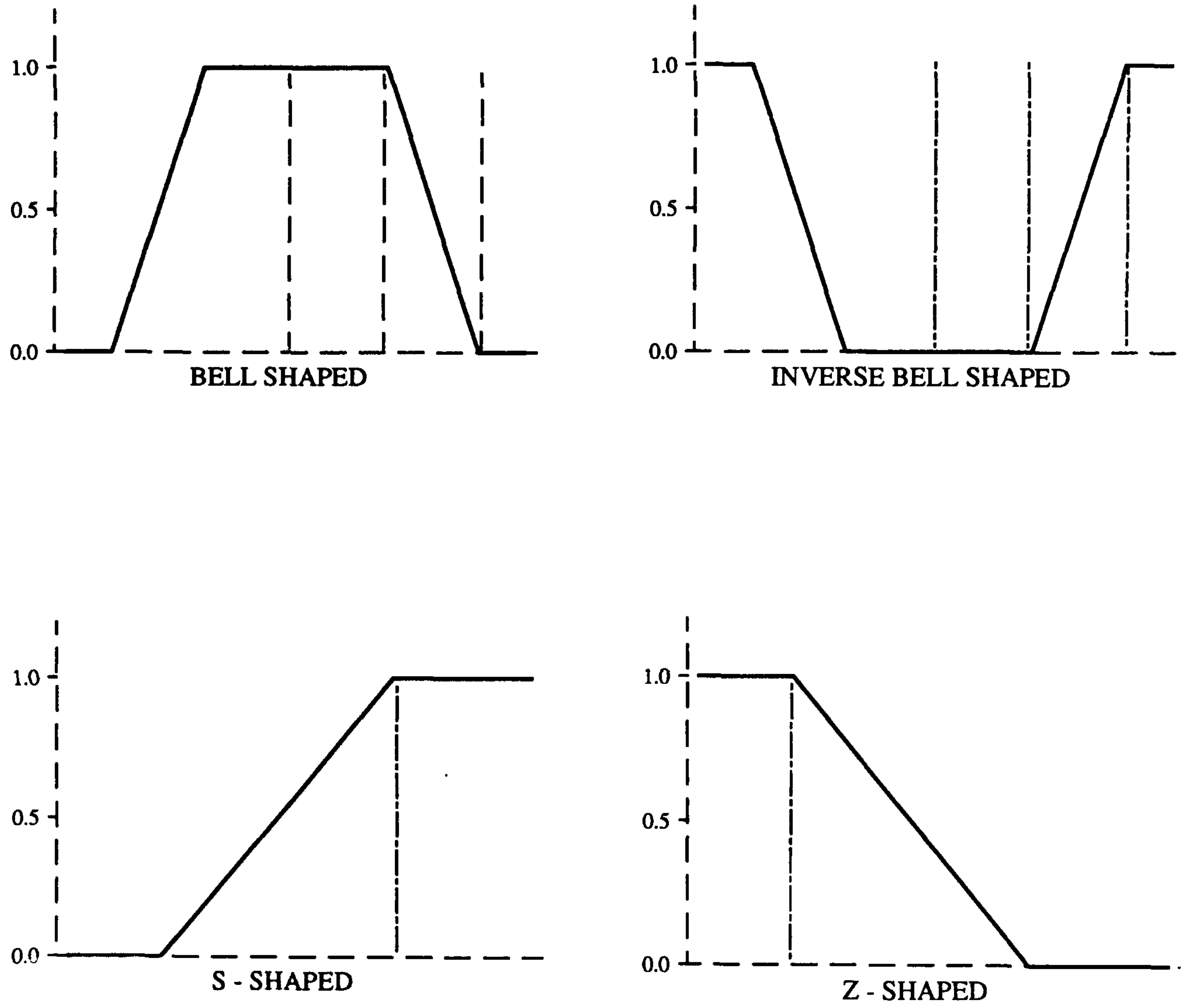

Figure 2: Common membership functions

for each potential observation of an attribute. The GCT-defined function generators, which were described above for specifying inexact values for attributes in the knowledge base, may also be used in translating an observed value into evidence, *i.e.*, a possibility distribution.

An explicitly defined interpretation for the pulse-width attribute requires a possibility distribution for each of its observable values. The format of such an interpretation consists of a description of the observations followed by a list or function description giving the possibility of each attribute value for a given observation. For example, an observation interpretation for pulse width may take the form

```
(HI-RANGE 3.97 LO-RANGE 2.08 OBS-TYPE FLOATING-POINT)
(PULSE-WIDTH  0.05 PEAK-WIDTH 0.02 DIST-TYPE NUMERIC)
```

This interpretation definition for the attribute pulse-width specifies that an observation is to be interpreted as the center point of a bell-shaped membership function that produces grades of membership for each value within the range 2.08 to 3.97 using the range-step 0.01 specified for pulse-width attribute. For example, if the observation for the pulse width is 2.22, then the possibility distribution that describes this information is

$$0.22/2.18 + 0.88/2.19 + 1/2.20 + 1/2.21 + 1/2.22 + 1/2.23 + 1/2.24 + 0.88/0.25 + 0.22/0.26.$$

4.1.3 Reasoning System

The Reasoning System consists of three major components: the Controller, the Evidential Interpreter, and the Support Generation System. The interaction among these three components is based on the processing cycle of observation acquisition by the Controller, transformation of observation into evidence by the Evidential Interpreter, and the generation of a measure of support for the domain object in the Support Generation System.

4.1.4 Controller

The Controller is responsible for managing the identification process. It assumes that the Database System has been used to open the domain knowledge base and the Observation-Interpretation system has been used to open the OI knowledge base. The user must select the reasoning paradigm. The Support Generation subsystem may process information using the Dempster-Shafer approach or several techniques utilizing fuzzy evidential reasoning. A separate subsystem is used for the probabilistic analysis.

The Reasoning System has three modes of execution: manual, automatic, and input. With manual execution, the user establishes the knowledge base object to be identified, the order and number of the attributes for which an observation is provided, and the observations themselves. This mode is primarily for testing and demonstration.

Automatic execution is the default mode. The controller randomly chooses an element from the knowledge base to be identified, and then determines the order of the attributes for which observation is to be acquired. The observation is also generated randomly about the actual value of the attribute of the selected object. This mode will be used for parametrically examining the effectiveness of the alternative support generation strategies.

The input mode of execution permits the use of a previously created script file as input. A script file contains all the information required from the user when execution mode is manual, except that it is stored in a file instead of being interactively provided. The script file may be created by recording the automatic or manual execution of the system. A script file may also be created using a text editor.

4.1.5 Evidential Interpreter

Once an observation is obtained, the Evidential Interpreter receives the information and transforms it into evidence. The transformation into evidence is defined by the OI knowledge base. For each attribute in the domain knowledge base, the OI knowledge base contains a mapping of each observation into a possibility distribution over the set of possible attribute values. The resulting possibility distribution is combined with the information in the domain knowledge base using the compatibility function.

4.1.6 Support Generation

The Support Generation System receives the evidence for an attribute in the form of a possibility distribution produced by the Evidential Interpreter. The choice of the reasoning paradigm dictates the representation of the evidence and the techniques used to update the measure of belief upon the acquisition of additional information.

For each object in domain knowledge base, a compatibility measure of the evidence and the partial membership function that describes the value of the attribute of the object (Section 3.1) is produced. Using the compatibility measure, the Support Generation System updates the support measure for all the elements in the frame of discernment using the appropriate combination function.

4.2 Domain Knowledge Bases

Domain information describing each element in the frame of discernment is contained in the domain knowledge base. Each entry describes an object in the frame of discernment using the fuzzy attribute-value representation. The Database System is used to define the frame structure and construct the elements of the knowledge base. In this section we discuss the internal representation of the domain knowledge base.

Domain objects in the domain knowledge base are represented as list structures. The knowledge base is a text file that can be used both by the programmers and the programs.

The domain objects in the sample radar knowledge base are types of emitters. The attributes of one of the entries, emitter type 1, ET1, are described in English in Example 2 (page 5). As can be seen in Figure 3, the function RANGE is used extensively to represent intervals. The RF attribute of emitter type 1 has a center "most observed" range, a "probable" range that includes it, and a "possible" range that includes them both. This is represented as five nonoverlapping ranges, the two outermost as possible, their inner neighbors as probable, and the center as the range most observed. For example,

```
((RANGE 2.30 2.54) LEAST-LIKELY)
```

describes the low end of the values compatible with the radar frequency of emitter type 1. `(RANGE 2.30 2.54)` describes the numerical limits of the range, and `LEAST-LIKELY` attaches the belief value "possible" to that range. The possibility values assigned to most-likely, probable, and least-likely are 1.0, 0.75 , and 0.5, respectively. The ranges of other attributes, such as PRI, are given a belief value of 1.0, that is, they are considered equally possible.

The prologue of a knowledge base contains the information that defines the properties of the elements of the knowledge base. The prologue for the radar knowledge base is given in Figure 4. The entry LEGALDB begins the prologue. This is followed by a list that specifies the name of the knowledge base, RADAR, and the names of the attributes. The remaining lists describe the properties of each attribute. For example, the values of attribute RF (radio frequency) are single values `(MULTI-VAL N)`, floating point `(DATA-TYPE 2)` and range from 3.97–2.08 GHz in steps of 1.

The pulse-repetition interval attribute, PRI, is described with three entries in the prologue. PRI is used for emitters that operate in constant PRI mode. PRI-ELEMENT1 and PRI-ELEMENT2 attributes are used for emitters that operate in staggered mode.

The List Data Structure	Explanation
`(ET1`	Emitter Type 1 Attributes
`(RF (LIST`	RF Details
`((RANGE 2.30 2.54) LEAST-LIKELY)`	possible: 2.30-3.47 GHz
`((RANGE 3.25 3.47) LEAST-LIKELY)`	
`((RANGE 2.54 2.77) PROBABLE)`	probable: 2.54-3.25
`((RANGE 3.01 3.25) PROBABLE)`	
`((RANGE 2.77 3.01) MOST-LIKELY))`	most observed: 2.77-3.00
	PRI Details
`PRI (LIST`	Constant PRI Mode
`((RANGE 90.96 101.28) 1.0)`	90.96 - 101.27 μsec
`((RANGE 116.87 133.63) 1.0)`	116.87 - 133.62
`((RANGE 179.19 199.22) 1.0))`	179.19 - 199.22
`PRI-ELEMENT1 (LIST`	Staggered PRI Mode
`((RANGE 66.42 71.12) 1.0)`	element 1a: 66.42–71.11 μsec
`((RANGE 74.76 107.91) 1.0))`	element 1b: 74.76–107.90
`PRI-ELEMENT2 (LIST`	
`((RANGE 136.36 157.58) 1.0)`	element 2a: 136.36–157.57
`((RANGE 191.08 194.09) 1.0))`	element 2b: 191.08–194.09
`PW`	
`((RANGE 0.2 0.3) 1.0))`	limits: 0.2–0.3 μsec.
`SCAN-PERIOD (LIST`	Scan Details
`((RANGE 0.04 0.06) 1.0)`	CON: 0.04–0.06 sec
`((RANGE 2.0 4.0) 1.0)))`	BDS: 2.0–4.00
`)`	

Figure 3: Database entry for emitter type 1

```
LEGALDB

(RADAR (RF PW PRI PRI-ELEMENT1 PRI-ELEMENT2 SCAN-PERIOD))

(RANGE-STEP 0.01 MULTI-VAL N
HI-RANGE 3.97 LO-RANGE 2.08 DATA-TYPE 2 NAME RF)

(RANGE-STEP 0.01 MULTI-VAL N
HI-RANGE 1.0 LO-RANGE 0.15 DATA-TYPE 2 NAME PW)

(RANGE-STEP 1.00 MULTI-VAL N
HI-RANGE 223.32 LO-RANGE 53.25 DATA-TYPE 2 NAME PRI)

(RANGE-STEP 1.00 MULTI-VAL N
HI-RANGE 122.8 LO-RANGE 53.25 DATA-TYPE 2 NAME PRI-ELEMENT1)

(RANGE-STEP 1.00 MULTI-VAL N
HI-RANGE 194.09 LO-RANGE 63.7 DATA-TYPE 2 NAME PRI-ELEMENT2)

(RANGE-STEP 0.01 MULTI-VAL N
HI-RANGE 10 LO-RANGE 0.04 DATA-TYPE 2 NAME SCAN-PERIOD)
```

Figure 4: Prologue for radar knowledge base

4.3 Radar identification example

The preceding sections described the domain and evidential representations and support generation techniques used by the GCT. In this section we illustrate the properties of the GCT by examining the identification process using the radar problem domain. The frame structure and representation for the radar knowledge base were described in the previous section. The complete knowledge base, consisting of six emitter types, is given in Appendix B. A possibility distribution over the six emitter types is constructed to represent the support based on the acquired information. The session shown in Figure 5 used the dynamic mode for object selection and observation acquisition.

Line 1 indicates that ET6, emitter type 6 is the radar type to be identified. The identification proceeds by acquiring information concerning the characteristics of the an attribute. Using the dynamic mode, the controller chooses an attribute and obtains a value for that attribute. Line 2 shows that the radio frequency was the chosen attribute and the observation indicates that the unknown emitter is operating at 3.06 GHz.

The evidential interpreter produces the compatibility measure for each of the emitter types based on the observation. This is transformed into a possibility distribution (because of the choice of possibilistic reasoning) which indicates a measure of support for each type based on the single observation. After processing each observation, the support for each of the emitter types based on the accumulated evidence is computed and output (lines 4–7). Emitter types that are determined to be incompatible with the evidence are omitted from the listing.

The identification continues with the acquisition of another observation (line 9). The pulse width is reported to be 0.19 μsec. Updating the support based on this evidence indicates that only emitter type 6 is consistent with the accumulated information (line 9). The acquisition of additional information only confirms emitter type 6. Another iteration chooses emitter type 4 as the unidentified object (line 24). This time, three observations are required to unambiguously identify the emitter type.

The rapid convergence to a definite conclusion in the preceding examples is a result of the makeup of the radar knowlege base. The domain consisted of only six emitter types with little overlap in the characteristics of the different types.

```
HYPOTHESIS DOMAIN OBJECT : ET6

OBSERVATION      RF:   3.06
DOMAIN OBJECT : ET3      SUPPORT: 1.0
DOMAIN OBJECT : ET6      SUPPORT: 1.0
DOMAIN OBJECT : ET1      SUPPORT: 0.75
DOMAIN OBJECT : ET5      SUPPORT: 0.75
 Press RETURN to continue
OBSERVATION      PW :  0.19
DOMAIN OBJECT : ET6      SUPPORT: 1.0
Press RETURN to continue
OBSERVATION      SCAN-PERIOD :  8.25
DOMAIN OBJECT : ET6      SUPPORT: 1.0
Press RETURN to continue
OBSERVATION      PRI-ELEMENT1 :   254.49
DOMAIN OBJECT : ET6      SUPPORT: 1.0
Press RETURN to continue
OBSERVATION      PRI-ELEMENT2 :   257.96
DOMAIN OBJECT : ET6      SUPPORT: 1.0
Press RETURN to continue
HYPOTHESIS DOMAIN OBJECT : ET4
OBSERVATION      RF :  2.33
DOMAIN OBJECT : ET4      SUPPORT: 0.75
DOMAIN OBJECT : ET1      SUPPORT: 0.5
DOMAIN OBJECT : ET3      SUPPORT: 0.5
Press RETURN to continue
OBSERVATION      PW :  0.66
DOMAIN OBJECT : ET4      SUPPORT: 0.75
DOMAIN OBJECT : ET3      SUPPORT: 0.5
Press RETURN to continue
OBSERVATION      SCAN-PERIOD :  0.05
DOMAIN OBJECT : ET4      SUPPORT: 0.75
Press RETURN to continue
OBSERVATION      PRI :  87.0
DOMAIN OBJECT : ET4      SUPPORT: 0.75
Press RETURN to continue
```

Figure 5: Radar identification example

5 Testing Methodology

The GCT has been developed to examine and compare the effectiveness of the evidential reasoning techniques under varying conditions of domain and evidential uncertainty. This section describes the knowledge bases and measurement criteria used in the analysis.

5.1 Domain Knowledge Bases

Five domain knowledge bases were created for the analysis. The knowledge bases differ by the distribution of the objects and the precision in the definition of the attributes. Each domain knowledge base has 100 objects. The objects in the knowledge bases are defined by three integer-valued attributes ATTR-1, ATTR-2, and ATTR-3 whose domains are

$$\begin{aligned} \text{ATTR-1} &= \{1,2,3,\ldots,98,99,100\} \\ \text{ATTR-2} &= \{1,2,\ldots,10\} \\ \text{ATTR-3} &= \{0,1\}. \end{aligned}$$

Domain information for attribute three is considered to be precise. Consequently, for every domain object, ATTR-3 is defined either by the membership function $1/0 + 0/1$ or by $0/0 + 1/1$. The domain information for attributes ATTR-1 and ATTR-2 is imprecise and represented by a bell-shaped partial membership function. A bell-shaped membership function is determined by the midpoint p and parameters α and β. The membership of attributes values in the interval $[p-\alpha,\ p+\alpha]$ is *one*. If α is increased, then a wider range of attribute values are considered completely compatible with the domain information. The β value determines the size of the focal set of the bell-shaped distribution. Section 4.1 contains a complete description of a bell-shaped distribution.

The domain knowledge bases are divided into two classes based on the distribution of the objects. In the uniform knowledge bases, the midpoints of the imprecise attributes are chosen uniformly from the possible values. Normal knowledge bases were constructed to simulate the clustering of domain objects. The knowledge bases will be referred to as precise-uniform, medium-uniform, high-uniform, medium-normal-two-cluster and medium-normal-five-cluster, respectively. The first word in the name—*precise, medium, high*—specifies the amount of imprecision in the partial membership functions that define the attributes. The second word—*uniform, normal*—describes the type of distribution used to generate the midpoints for the imprecise attributes.

Values for attributes one and two for domain objects in the set of uniform knowledge bases are defined by midpoints p_1 and p_2, respectively. These values are randomly

chosen from a uniform distribution over the domain of the attribute. The precise value for attribute three was obtained by a uniform random selection from the set $\{0, 1\}$.

The three uniform domain knowledge bases are produced from the same randomly generated points. These knowledge bases differ only in the precision of the representation of domain information. This difference is obtained varying the α and β values for attributes one and two. In all three domain knowledge bases, membership functions defining ATTR-3 are assumed to be precise. The parameters defining the uniform knowledge bases are given in the table below.

	ATTR-1		ATTR-2		ATTR-3
DOMAIN DATA BASE	α	β	α	β	
precise-uniform	0	1	0	1	(precise)
medium-uniform	5	10	0	1	(precise)
high-uniform	10	20	1	4	(precise)

The clustered knowledge bases are created in two steps. First, the IMSL statistics library, a collection of FORTRAN statistical analysis routines, was used to generate 100 multivariate values. The covariance between the pairs of components was specified as zero. The first component of the multivariate value has a normal distribution with mean zero and variance 10.0, denoted $N(0, 10.0)$, where the second component has a $N(0, 1.0)$ *distribution. The third component has a* $N(0, 1.0)$ distribution. For each multivariate value, a value u from the uniform distribution over $[0, 1]$ was also generated. This value is used to determine in which cluster to place an object. In the second step, clusters were created with respect to ATTR-1 and ATTR-2 by transforming the first two components of each of the 100 multivariate values through the formula for the contaminated multivariate normal distribution [4] with m the number of components in the multivariate value:

$$pN_m(\mu_1, \sigma_1) + (1 - p)N_m(\mu_2, \sigma_2)$$

The addition in this formula refers to combining the process of realizing an object from a bimodal distribution. With probability p, an object is realized from $N_m(\mu_1, \sigma_1)$ and with probability $(1 - p)$ from $N_m(\mu_2, \sigma_2)$.

The parameters for creating the two-cluster domain knowledge base are as follows:

$$\begin{aligned} m &= 2 \\ p_1 &= 0.6 \\ p_2 &= (1 - p_1) = 0.4 \\ \mu_1 &= [63.0, 6.0] \\ \mu_2 &= [33.0, 3.0] \end{aligned}$$

$$\sigma_1 = \sigma_2 = ... = \sigma_5 = \begin{bmatrix} 10.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix}$$

The third component of the multivariate value was not considered in the clustering since it mapped to ATTR-3, with a precise binary valued attribute. There would be no possibility of overlap between the clusters if all objects in cluster one had the same value for ATTR-3, and this value was different from the value for all objects in cluster two. The objects in the two-cluster domain knowledge base were created as follows:

1. The uniform number associated with the generated multivariate value was tested. If $u \leq p_1$, then the first and second components of the multivariate value were added to the respective components of μ_1.

2. If $u > p_1$, then the first and second components of the multivariate value were added to the respective components of u_2.

3. Each component in the addition result was then rounded to the nearest integer value and taken to be the respective values for ATTR-1 and ATTR-2

4. The value for attribute ATTR-3 was set to zero if the third component of the multivariate value was less than or equal to zero; otherwise, it was set to one.

5. The attribute values generated were then fuzzified using the bell-shaped function and the parameters used for the medium-uniform knowledge base.

Creation of the five-cluster domain knowledge base followed the above procedure but five distributions instead of two were mixed. The parameters for creating the five-cluster domain knowledge base are as follows:

$$
\begin{aligned}
&m = 2 \\
&p_1 = 0.2 \quad \mu_1 = [20.0, 1.0] \\
&p_2 = 0.3 \quad \mu_2 = [35.0, 3.0] \\
&p_3 = 0.1 \quad \mu_3 = [55.0, 5.0] \\
&p_4 = 0.1 \quad \mu_4 = [75.0, 7.0] \\
&p_5 = 0.3 \quad \mu_5 = [95.0, 9.0] \\
&\sigma_1 = \sigma_2 = ... = \sigma_5 = \begin{bmatrix} 10.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix}
\end{aligned}
$$

1. The uniform number u associated with the generated multivariate value was examined. The cluster to which this object is assigned is determined by comparing this value to the p_i's. The first and second components of the multivariate value were added to the respective components of the appropriate μ_i.

2. Each component in the addition result was then rounded to the nearest integer value and taken to be the respective values for ATTR-1 and ATTR-2. Some of the clusters, *i.e.*, cluster one and cluster five have mean values for attributes that

are at the low or high range for its possible values. If the generation produced a value that was outside the legal range, the object still received that value for the attribute.

3. The value for attribute ATTR-3 was set to zero if the third component of the multivariate value was less than or equal to zero; otherwise, it was set to one.

4. The attribute values generated were then fuzzified using the bell-shaped function and the parameters from the medium-uniform knowledge base.

5.2 Observation and Interpretation

The identification process is driven by the acquisition and interpretation of information describing the properties of the unknown object. In the GCT, two components combine to produce the evidential distributions. A user defined probability distribution defines the accuracy of the observation, the proximity of the randomly produced center point to that of the actual center point of the object being identified. The second step in the construction of the evidential distribution is the interpretation of the observation. This introduces imprecision into the observation. An observation-interpretation knowledge base contains the information to transform the observation e into the evidential distribution Π_e.

Two probability distributions, P_{medium} and P_{high}, were used to analyze the effects of varying the inaccuracy of the observations. An observation is considered accurate if it agrees closely with the actual midpoint that defines the attribute of the selected hypothesis. The parameters used to generate P_{medium} and P_{high} are summarized in the table below.

	ATTR-1 distribution		ATTR-2 distribution		ATTR-3 distribution	
Observations	Type	Std Dev	Type	Std Dev	Type	Std Dev
P_{medium}	Normal	5	Normal	0.5	Uniform	0.1
P_{high}	Normal	10	Normal	1	Uniform	0.1

P_{high} is similar to P_{medium} except that the standard deviations for ATTR-1 and ATTR-2 are greater. That is, for P_{high} the observations generated by the GCT controller may be farther from the actual attribute values of the object selected for identification. In both cases an observation of ATTR-3 was generated by returning the actual value with probability 0.99.

Three different evidential interpretations were defined by three observation-interpretation knowledge bases in order to test the effects of evidential imprecision in

generation of support. The precise evidential interpretation treats the observed values for all attributes as precise information. Thus, an observed value x is represented by the possibility distribution $1/x$. For the medium evidential interpretation observations for attribute ATTR-1 are defined by the bell-shaped distribution function above, with $\alpha = 5$ and $\beta = 10$. For attribute ATTR-2, the bell-shaped distribution function with $\alpha = 0$ and $\beta = 2$ is used. For the high evidential interpretation observations for attribute ATTR-1 are defined by the bell-shaped distribution function with $\alpha = 10$ and $\beta = 15$. For attribute ATTR-2, the bell-shaped distribution function with $\alpha = 1$ and $\beta = 4$ is used. Observations for ATTR-3 are considered to be precise in all three evidential interpretations. For example, an observation of 0 for ATTR-3 is simply represented by the possibility distribution $1/0$.

In the tests described in the following section, we have used the bell-shaped distribution for both representations. Other distributions may be used for fuzzifying the information represented in the domain and evidential knowledge bases. To generate observations for an attribute, the GCT references the user-defined probability distribution specified for that attribute in the OI knowledge base. For example, for an attribute, a normal distribution with a standard deviation of 3.0 might be specified. The mean is the actual domain knowledge base value for the attribute of the selected hypothesis. The user-defined probability distributions used to generate observations may be changed easily. The ability to control the precision of the observations used by an evidential reasoning technique permits determining its practical limitations when faced with increasingly inaccurate observations.

5.3 Testing Procedure

A *test* consists in selecting an object at random from the domain knowledge base, generating observations describing the selected object, and applying the support generation and updating strategies to construct a measure that estimates the likelihood of the elements in the frame of discernment based on the observations. A *test run* is the result of iterating individual tests. For each test run, a number of parameters must be specified:

- the domain knowledge base
- the OI knowledge base
- the evidential reasoning paradigm
- the number of observations of one hypothesis
- the number of tests per test run

The domain knowledge base determines the degree of uncertainty in the domain knowledge. The OI knowledge base specifies the degree of imprecision to be introduced into the generated observations. In addition, the number of tests and the number of observations for each test must be specified.

For statistical analysis, each test run consisted of 100 tests. For all the test runs, the number of observations per test was ten. Table 1 summarizes the test runs and identifies each with a number. A value in the Domain column and the Interpretation column represents the the degree of fuzziness present in the appropriate knowledge base for the corresponding test run labelled by the row. Also, the two-cluster and five-cluster domain knowledge bases are prefixed with "2c-" and "5c-" respectively. A value in the Observation column represents the degree of variability of a generated observation from the value found in the domain knowledge base.

5.4 Measurements and Analysis Techniques

The measures used to compare the effectiveness of these evidential reasoning techniques are determined from the support assigned to the objects at the completion of the test. The level-n support and the entropy along with their means and standard deviations are calculated for each test run. As described in Section 3.3, the plausibility of singleton sets is used to construct the ranking of candidates when the D-S technique is employed.

A test has *level-n* support if the number of objects that have support greater than or equal to that of the selected object is n or less. For example if three objects, one of which is the selected object, are tied with the highest level of support, the test has level-3 support. If five objects tied with the highest level of support and the selected object has the next highest support (without ties), the support level is 6. Level-n support measures the extent to which the reasoning method assigns support to the selected object. However, it does not reflect the extent of rejection of other objects. For that, we turn to entropy.

The *entropy* or information expectation is a measure of the uncertainty present in a probability or possibility distribution. For an m-point discrete probability distribution $\{p(x_i) \mid 1 \leq i \leq m\}$ the entropy is

$$-\sum_{i=1}^{m} p(x_i) \log_2 p(x_i)$$

and ranges from zero (certainty) to $\log_2 m$ (maximum uncertainty). The entropy value is a measure of the certainty of the final support to hypotheses regardless of the selected hypothesis.

Table 1: Test case summary

Case	Domain	Interpretation	Observation
1	none	none	medium
2	none	medium	medium
3	none	high	medium
4	medium	none	medium
5	medium	medium	medium
6	medium	high	medium
7	high	none	medium
8	high	medium	medium
9	high	high	medium
10	2c-medium	none	medium
11	2c-medium	medium	medium
12	2c-medium	high	medium
13	5c-medium	none	medium
14	5c-medium	medium	medium
15	5c-medium	high	medium
16	none	none	high
17	none	medium	high
18	none	high	high
19	medium	none	high
20	medium	medium	high
21	medium	high	high
22	high	none	high
23	high	medium	high
24	high	high	high
25	2c-medium	none	high
26	2c-medium	medium	high
27	2c-medium	high	high
28	5c-medium	none	high
29	5c-medium	medium	high
30	5c-medium	high	high

6 Analysis of Techniques

In this section the effectiveness and the efficiency of the support generation paradigms are compared over the series of test domains summarized in Table 1. The measures by which the effectiveness of the reasoning algorithms are compared are the level-n support and the entropy. The level support data are shown as frequency graphs in Appendix A. The mean entropy and standard deviation of the support distributions generated by the tests Table 2.

The graphs in Appendix A show the frequency of level-n support. The features of interest in studying the level support are the level-1 support, the maximum frequency, and the rate at which the curve rises from the level-1 support toward the frequency of 100. The level-1 support measures how often the selected object was ranked higher than all other objects. The graph of a level-n *vs* frequency is an increasing function since a test that is level-i is also level-$i+1$. The difference between the number of tests and the maximum frequency specifies the number of times that the selected object was given no support.

On graph 2 (obtained from the test run using precise domain information, medium evidential imprecision and medium variance) the level-1 support for the probabilistic method is approximately 55. That signifies that in 55 of 100 tests, the selected object was given the most of the probabilistic support. The probabilistic curve in graph 2 rises to frequency of about 82 at level-5 support. This signifies that the target object was in the top 5 of supported objects 82 times out of the 100 runs. The curve rises little thereafter with increasing percentages and ends at support level 6. No values for higher percentages are shown. This indicates that the probabilistic method assigned some support to the selected object 82 times in the 100 tests. In the other cases, the selected object received no support.

The entropy value indicates the degree of uncertainty in the final support distribution. The entropy measure assumes values between zero and $\log_2 100$. The maximum uncertainty occurs when no object is favored. In this case the entropy is $\log_2 100$, which is approximately 6.64. An entropy of zero indicates a precise distribution, one in which a single object receives all the support.

Under certain combinations of domain representation and evidential uncertainty, a reasoning technique may fail to assign support to any of the domain objects. When this occurs the selected object is said to be eliminated from consideration. The entropy value is calculated only for the cases in which the reasoning system supports the selected object. Case 16 demonstrates the necessity of combining both the entropy and the level-n support when considering the effectiveness in a test scenario. The entropy of zero indicates that the resulting distributions select a unique object. However the level-n graph shows that this scenario, precise domain data and precise evidence, eliminates the selected object 99% of the time.

Table 2: Entropy results: mean and standard deviation

Case	Sup-Min		Exp Value		Prob'c		D-S	
	E	$S_{\overline{E}}$	E	$S_{\overline{E}}$	E	$S_{\overline{E}}$	E	$S_{\overline{E}}$
1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	0.86	0.77	0.84	0.76	0.83	0.78	0.57	0.61
3	1.91	0.79	1.94	0.80	1.58	0.84	1.28	0.79
4	0.87	0.77	0.87	0.77	0.73	0.71	0.74	0.72
5	2.32	0.71	1.93	0.73	1.55	0.72	1.28	0.77
6	3.09	0.59	2.73	0.59	2.40	0.65	1.79	0.75
7	2.74	0.59	2.74	0.59	2.39	0.68	2.29	0.63
8	4.12	0.50	3.70	0.53	2.60	0.62	2.36	0.64
9	3.64	0.60	3.25	0.56	2.95	0.53	2.65	0.63
10	2.90	1.18	2.90	1.18	2.50	1.20	2.66	1.14
11	4.34	0.69	4.09	0.74	3.27	0.92	3.52	0.81
12	4.62	0.50	4.51	0.56	3.54	0.59		
13	2.08	1.09	2.08	1.09	1.77	1.03	1.84	1.02
14	3.38	0.88	3.06	0.91	2.54	0.88	2.41	0.90
15	3.92	0.75	3.64	0.81	3.19	0.83		
16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
17	0.85	0.70	0.82	0.70	0.61	0.68	0.51	0.59
18	1.57	0.84	1.58	0.85	1.12	0.95	1.10	0.78
19	0.91	0.69	0.91	0.69	0.65	0.69	0.76	0.69
20	1.88	0.79	1.56	0.79	1.17	0.81	1.20	0.73
21	2.62	0.67	2.25	0.70	1.92	0.88	1.66	0.73
22	2.25	0.75	2.25	0.75	1.91	0.82	1.90	0.75
23	3.80	0.54	3.40	0.56	2.36	0.64	2.19	0.63
24	3.24	0.60	2.84	0.60	2.84	0.55		
25	2.49	1.37	2.49	1.37	1.64	1.38	2.30	1.36
26	3.77	0.99	3.42	1.06	2.44	1.08	3.09	1.03
27	4.36	0.71	4.11	0.80	3.13	0.90	3.68	0.84
28	2.08	1.19	2.08	1.19	1.40	1.17	1.84	1.17
29	2.95	1.11	2.60	1.10	1.94	1.11	2.23	0.95
30	3.55	0.94	3.20	1.00	2.68	1.16	2.68	0.99

The analysis begins by considering the effects of each of the parameters individually. This is followed by a discussion of the performance of the particular reasoning techniques: fuzzy evaluation and probabilistic. Finally the performance of the techniques is compared.

6.1 Evidential Precision

The capabilities of a sensor are defined by two properties, its accuracy and its precision. The accuracy refers to the proximity of the value returned by the sensor to the actual stimulus. The precision is the interpretation of the value. For example, a faulty scale may determine weight within plus or minus five pounds. This is the accuracy of the scale, the value returned is within 5 pounds of the true weight. Confidence in the value is the precision of the sensor. We will now examine the effects of the interpretation of sensor data on support generation. This will be followed by analysis of sensor accuracy.

The precision of an observation is determined by the interpretation of the sensor data. The incorporation of imprecision into the evidence is the result of the fuzzification of the data returned by the sensor. In the GCT, the degree of imprecision is determined by the parameters α and β that define the bell-shaped function. Three interpretations are used to determine the effect of evidential imprecision on support generation. The analysis of evidential imprecision begins by considering the case in which the data provided by the sensor is assumed to be precise.

The assumption that evidence is precise imposes restrictions on the ability of the reasoning system to correctly identify the selected object. Precise evidence concerning an attribute *at* is represented by a possibility distribution of the form $\Pi_e = 1/v_i$ where v_i is the value returned by the sensor. In the case of the GCT analysis, v_i is the value that is randomly generated based on midpoint of the partial membership function defining the attribute *at* of the selected object and the accuracy assigned to the sensor.

A domain object is eliminated from consideration if the evidence is incompatible with the domain information. For an object h to receive support from precise evidence, the value v_i must fall with the range of values deemed consistent with h by the membership function $at(h)$. When this does not occur, h is assigned a zero likelihood and removed from further consideration by all of the reasoning techniques. Table 3 shows the number of times the selected object is eliminated by the limitations imposed by precise information. The data in Table 3 are averages of the results over the four evidential support generation strategies.

When the domain knowledge is also considered precise, an object receives support only when domain data and the evidence are an exact match. Over a number of observations from an inaccurate sensor, at least one observation may be expected to

Table 3: Precise evidence

Knowledge Base	Percent Eliminated medium variance	Percent Eliminated high variance	Level-3 Support medium variance	Level-3 Support high variance
precise-uniform	99	99	1	1
medium-uniform	21	80	70	48
high-uniform	3	25	65	47

vary from the precisely specified domain data. Our results indicate this to be the case, with 99% of the tests excluding the selected object. In general, the assumptions of precise domain and sensor data are unrealistic for evidential reasoning in an uncertain domain. This remark is supported by the poor performance of the reasoning paradigms under these conditions.

Imprecise domain information increases the likelihood of a match with precise evidence. Thus the highly fuzzified domain knowledge base only eliminates the selected object on three percent of the tests when the accuracy of the sensor is defined by the medium variance. Increasing the variance, or equivalently decreasing the simulated accuracy of the sensor, exacerbates the poor performance when the evidence is considered precise.

The final two columns of Table 3 illustrate the discriminatory capabilities of using precise evidence when the selected object receives support. The columns indicate frequency at which the selected object receives level-3 support or less. With the medium-uniform knowledge base, the selected object has level-3 support 87% of the time when it is not completely eliminated. With the high-uniform knowledge base, the corresponding level-3 support is 63%. These results show the expected relationship that increasing the domain imprecision decreases the ability of the reasoning paradigms to discriminate among the objects.

The level-3 support results of the tests using the probabilistic support are given in Table 4. These show that adding imprecision to the evidential interpretation improves the performance over that of an precise interpretation. The fuzzification allows the evidence to overlap the domain knowledge representation so that the selected object receives support. However, excessive imprecision weakens the ability of the paradigm to focus on the selected individual. In all the tests except those with precise domain knowledge and high inaccuracy, the tests in which medium evidential imprecision was used outperformed both the precise and the high imprecision scenarios.

Table 4: Effects of evidential imprecision

	Level-3 Support					
	Medium Variance			High Variance		
Evidential Interpretation	Precise KB	Medium KB	High KB	Precise KB	Medium KB	High KB
precise	1	76	73	1	18	52
medium	77	92	90	17	61	71
high	56	66	68	33	47	58

6.2 Sensor Accuracy

Inaccuracy measures the distance of the value returned by a sensor to the actual value of the object being scrutinized. In the GCT, the degree of inaccuracy is determined by the probability distribution used to generate the observation. An observation concerning an attribute *at* is generated from a normal distribution centered at the midpoint of the partial membership function defining the attribute *at* of the selected object. As the variance of this normal distribution increases, the inaccuracy in the observations also increases. Two levels of accuracy, *medium variance* and *high variance*, were used in this study. The associated probability distributions are given in Section 5.2.

Increasing the inaccuracy has the same effect as reducing the precision in evidence, it reduces the likelihood of the overlap of the focal set evidential distribution with that of the domain information. When this occurs, the selected hypothesis is assigned no support and is removed from further consideration by all reasoning techniques. Table 5 gives the number of times the selected object was eliminated in the indicated scenario. This value is the average over the four evidential support generation techniques. Increasing the inaccuracy causes a greater number of tests to reject the selected object. When the accuracy was determined by the medium variance distribution, only once did the selected object fail to receive support in the medium-uniform knowledge base and medium evidential interpretation case. This grew to 25% of the tests with the greater inaccuracy. As with precise data, increasing the imprecision in the domain knowledge mitigates the error and reduces the elimination. The same phenomena occurs by increasing the evidential imprecision. This expands the focal set of the evidential distribution increasing the likelihood of a non-null intersection with the domain representation.

The accuracy also affects the level of support for the selected object. Increasing the inaccuracy significantly decreases the number of times the selected hypothesis receives level-3 support. This is exhibited by Table 4, which displays the level-3 support of the

Table 5: Effects of inaccurate observations

Evidential interpretation	Percent Eliminated: Medium Variance, Precise KB	Medium Variance, Medium KB	Medium Variance, High KB	High Variance, Precise KB	High Variance, Medium KB	High Variance, High KB
precise	99	21	3	99	80	25
medium	20	1	1	80	25	5
high	8	1	1	55	10	3

probabilistic tests. For the medium-uniform knowledge base, decreasing the accuracy reduces the average level-3 support from 92% to 61%. As the imprecision in the evidential interpretation increases, the difference in the level-3 support for medium variance and high variance are still significant but become smaller.

6.3 Imprecision of Domain Information

Imprecision in the representation of domain information is indicated by the breadth of the partial membership functions defining the characteristics of the objects. This, in turn, is determined by parameters of the bell shaped membership functions at_1 and at_2. Knowledge bases were built with precise, medium, and high imprecision to study the effects of altering of domain precision on the effectiveness of support generation. The parameters defining these knowledge bases are given in Section 5.1. The level-n support for the precise knowledge bases are given in Appendix A in graphs

Precise-Uniform: Graphs 1, 2, 3, 16, 17, and 18.

Medium-Uniform: Graphs 4, 5, 6, 19, 20, and 21 are medium-uniform, and Graphs 10-15 are medium-normal-two-cluster and 25-30 are medium-normal-five-cluster.

High-Uniform: Graphs 7, 8, 9. 22, 23, and 24.

Altering the precision of the domain representation has effects that are similar to those observed when changing the evidential precision. The consequences of precise domain knowledge has already been observed in the analysis of sensor accuracy and precision. When the evidence is either precise or highly inaccurate, the likelihood of obtaining an observation that does not support the selected is high. The selected object is the assigned zero support and removed from consideration. Graphs 1 and 16, the combination of precise domain information and precise evidence, show the nearly total elimination of the selected object.

Increasing the imprecision in the domain representations raises the ikelihood of a partial match of the evidence with the membership function defining the attribute

of the selected object. In graph 3, an exception, the observation and high evidential interpretation have compensated markedly for the precision of the knowledge base. Retention using the medium-uniform knowledge base is improved over precise-uniform. Graphs 4 and 7 show the effects of increasing the domain imprecision with precise evidence. The rate at which the selected object is eliminated drops from 99% with precise knowledge representation to 20% with medium and 2% with high.

The same trend occurs, but to a lesser degree, when the evidence is imprecise. Combining precise domain knowledge with intermediate evidential interpretation and low inaccuracy produces elimination rates of 16% and 25% (graphs 2 and 20). The elimination is neglegible when the medium knowledge bases are employed. With the higher sensor inaccuracy, elimination occurs with the medium knowledge bases. The highly imprecise domain knowledge base is required to overcome the elimination caused by the data.

The preceding observations indicate that the rate elimination can be improved by decreasing the precision of the knowledge base. A fuzzification of the domain knowledge, however, reduces the discriminatory capability. In the extreme case, we would have a completely ambiguous knowledge base guaranteed to capture all objects. Discrimination is quantified by the entropy of the support distribution. To illustrate the reduction of the support, we consider the case with uniform knowledge base, medium sensor inaccuracy, medium evidential interpretation (tests 2, 5, and 8) and the probabilistic approach. The entropy for the precise knowledge base is 0.83. This rises to 1.5 and 2.6 for the more imprecise knowledge.

6.4 Fuzzy Techniques

The analysis included two support generation techniques based on fuzzy reasoning. These techniques differed in the initial phase of the support generation, the determination of compatibility of domain information with evidence. Sup-min composition and fuzzy expected value operator (Section 3.1) are used to build the distribution $\Pi(e)$.

Sup-min composition chooses the maximal pointwise agreement between the domain information and the evidence. With the expected-value operator, the compatibility between the evidence and domain information is determined by the overlap of their respective possibility distributions.

Regardless of the inaccuracy of the observations, for precise evidential interpretations, the test results show that these two techniques perform identically. Since a precise evidential interpretation of an observation v produces the possibility distribution $1/v$, the expected-value operator computes the same compatibility as the sup-min composition.

In all tests with medium and high evidential interpretations, the expected-value

technique produces higher level-3 support. Increasing the imprecision in the evidential interpretation increases the likelihood of overlap of the focal elements in the evidential possibility distribution with that of the domain information. Since the sup-min uses the possibility value of the single point in the intersection of the focal points of the two distributions, it ignores the support assigned by the evidence that does not agree with the domain information. The expected-value method is one of averaging, that is, it incorporates the entire focal set of the evidence and tends to produce more discriminatory information.

The normalized entropies for expected-value are also lower than those for the sup-min. As inaccuracy increases in the observations, the spread between the entropy measures widens. Thus, the support generated by expected value is less evenly distributed, *i.e.*, less ambiguous.

An exception to the superiority of the expected-value technique occurs for precise domain information and the high evidential interpretation. For these test runs sup-min produces a higher level-3 support but also a higher entropy. Since precise domain information for an attribute value is specified by a partial membership function $1/v$, the intersection of the domain information and the evidential possibility distribution contains at most a single focal element. Scaling by the sigma-count in the expected-operator reduces the compatibility measure between the two distributions. But the effects of this reduction do not appear until the high evidential interpretation is used.

6.5 Probabilistic Techniques

The remaining two reasoning paradigms generated support by probabilistic techniques. Updating in the D-S system used Dempster's rule; the strictly probabilistic analysis used Bayes' rule. With respect to level-3 support and rate of selected object elimination, the Dempster-Shafer technique outperformed the probabilistic in the test runs for high evidential imprecision regardless of the accuracy. It had the lower entropy measure for the test runs when there was medium variance in the accuracy of the observations. With high variance, however, the probabilistic had nearly equivalent entropy for the precise domain information and the high evidential interpretation.

The only test runs in which the probabilistic technique performed better with respect to the entropy measure was when the clustered domain knowledge bases were used. For high variance tests, the probabilistic approach has lower entropy values. For the non-clustered domain knowledge with high sensor inaccuracy, the entropy measures are nearly identical when either the domain information or evidential interpretation is precise or when both the domain information and evidential interpretation is medium.

An important consideration between the probabilistic techniques is the resources required for the generation of support. The D-S theory, using a set based approach,

has exponential complexity with respect to the size of the focal sets. It is for this reason that there are no D-S results for the high imprecision domain knowledge with the highly imprecise evidential interpretation. This combination produces a large focal set in the evidential basic probability assignment. Thus in large or imprecise domains, the D-S approach is impractical without employing further simplifying assumptions on the relationship between evidence and domain knowledge.

6.6 U-uncertainty

The discussion in the preceding section used entropy to measure the uncertainty in the final assignment of support to the domain elements. Historically, entropy is a measure designed for analyzing probability distributions. Since the fuzzy reasoning techniques produce possibility distributions, the uncertainty in the results are also analyzed using the corresponding possibilistic measure, U-uncertainty [5]. U-uncertainty is a measure of nonspecificity derived from a generalization of the Hartley information measure. U-uncertainty, which measures the uncertainty associated with a choice among a specific number of alternatives, is obtained by

$$U(\mathbf{r}) = \sum_{i=1}^{n} (\rho_i - \rho_{i+1}) \log_2 i$$

where $\mathbf{r}$ is a normal possibility distribution with n focal elements. The distribution is assumed to be sorted in descending order of possibility values

$$\{1.0 = \rho_1 \geq \rho_2 \geq \ldots \geq \rho_i \geq \rho_{i+1} \geq \ldots \geq \rho_n \geq 0\},$$

where $\rho_{n+1} = 0$ by convention.

The minimum of U($\mathbf{r}$) is zero and occurs when exactly one component of $\mathbf{r}$ is assigned possibility 1 and the value assigned to all of the remaining components is 0. That is, when the possibility distribution uniquely determines an object. The maximum of U($\mathbf{r}$) occurs when all elements have possibility 1, in which case

$$U(\mathbf{r}) = \log_2 n$$

where n is the size of the focal set of the distribution.

The U-uncertainty is computed using a normalized possibility distribution of the final support list. The measure is obtained from the probability distributions created by the probabilistic approach and the plausibility values generated by the Dempster-Shafer technique by linearly scaling the distributions so that they have a maximum value of one. The average and standard deviation were calculated for each test case. The result of a single iteration was used for average calculation only when the preselected object for identification was given nonzero support.

Table 6: Nonspecificity statistics U for reasoning techniques

Case	Sup-Min		Exp Value		Prob'c		D-S	
	$\overline{U}$	$S_{\overline{U}}$	$\overline{U}$	$S_{\overline{U}}$	$\overline{U}$	$S_{\overline{U}}$	$\overline{U}$	$S_{\overline{U}}$
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.71	0.72	0.69	0.69	0.60	0.64	0.39	0.54
3	1.49	0.79	1.77	0.84	1.33	0.73	1.02	0.84
4	0.71	0.72	0.71	0.72	0.51	0.62	0.54	0.62
5	1.95	0.81	1.21	0.70	0.83	0.57	0.68	0.59
6	2.39	0.62	1.83	0.64	1.36	0.53	0.94	0.63
7	2.14	0.65	2.14	0.65	1.57	0.77	1.62	0.72
8	2.91	0.56	2.29	0.57	1.56	0.67	1.39	0.62
9	3.64	0.52	2.58	0.50	1.55	0.55	1.53	0.63

Table 6 shows the results of this uncertainty measure for the first nine test cases. The U-uncertainty is zero for case 1 since only one iteration did not eliminate the selected object. In this case, the selected object was identified with certainty.

The trends in the U-uncertainty follow those observed in the entropy results. The Dempster-Shafer technique generally has the lowest values of the four techniques. The results of the Demspter-Shafer and probabilistic analysis show little difference with the precise evidential interpretations, cases 1, 4, and 7. The probabilistic technique has the next lowest U-uncertainty values but the values for the precise domain knowledge cases, 1 through 3 are not much lower than those for the other two techniques. For the probabilistic technique with high imprecision domain knowledge bases, cases 7–9, the U-uncertainty values are approximately equal regardless of the imprecision in the evidential interpretation. The U-uncertainty results indicate that the expected value technique has a greater discriminatory capability than the sup-min evaluation.

6.7 Comparison

The four reasoning techniques have been tested against various levels of sensor accuracy and imprecision in domain knowledge and evidential interpretation. Altogether thirty scenarios were examined, with widely differing effectiveness of the techniques as a function of the parameters. For a given scenario, however, the level-n graphs often are similar. Even in these cases, the entropy provides a method of distinguishing between the techniques.

The Dempster-Shafer technique, the fuzzy expected value technique, and the fuzzy sup-min technique produced level-n *vs.* frequency graphs of similar shape, displaced from one another roughly by the differences in their one percent ranking frequencies.

The curves that resulted from the three techniques are similarly convex and seldom intersect. Hence, the technique that had the best level-1 frequency was often the best technique for that scenario.

Two features incorporated into the GCT simulations may have advantageously effected the performance of the probabilistic techniques. The Dempster-Shafer combination rule requires that the basic probability assignments generated from the observations be independent. The Bayesian approach assumed conditional independence of evidence given the domain information. Since observations were generated from randomly selected objects, these assumptions were satisfied in the test domain knowledge bases. The support updating rule of the fuzzy strategies is a conservative approach. The use of the minimum operator is equivalent to assuming maximal correlation. In most problem domains, evidence is neither independent nor conditionally independent. In domains in which independence is present, the conservative updating rule of the fuzzy approach may inhibit the reduction of support assigned to objects that only partially match the evidence.

The domain knowledge bases constructed for this study also assumed no correlation between the values of the attributes for domain objects. This is the result of choosing the midpoints of the attributes defining the domain objects independently from random distributions. Lack of correlation between attributes is also rare in naturally occurring problem domains.

When comparing support generated for the selected object, the probabilistic techniques (D-S and Bayesian) preformed marginally better than the expected value fuzzy approach. Because of the conservative support generation technique, the sup-min composition produced lower support levels in the baseline domains. This was especially apparent in graph 16, where the level-3 support is only 7%.

The entropy, however, can still discriminate between the certainty in the support distributions. The probabilistic techniques have lower entropy values than the fuzzy. This indicates a more precise identification by those algorithms. This, again, is attributable to the conservative support updating in the fuzzy techniques. The proximity of domain objects affects the certainty of the support. In the uniform knowledge bases, D-S produces results with the least entropy. In the clustered domains, this difference disappears and is reversed in several cases.

The degree of clustering also affects the certainty of the result. The 2-cluster knowledge bases have higher entropy than the corresponding uniform knowledge bases. The proximity of a large number of similar domain objects spreads the support among them, which increases the uncertainty. The 5-cluster reverses this trend. The larger number of clusters insures that 80% of the objects are separated from the selected object. Thus the support is spread over fewer objects.

Although the Dempster-Shafer technique appears to be one of the most effective procedures, it is also the most resource intensive. For the baseline scenario, medium

domain imprecision, medium evidential interpretation, and medium sensor accuracy, the sup-min evaluation required 8,930 seconds, the expected value evaluation required 8,730 seconds, and the D-S 37,342 seconds. Dempster's rule, the updating procedure for the D-S approach is an exponential algorithm. The fuzzy techniques are both linear in the size of the frame of discernment. The Bayesian probabilistic approach, utilizing matrix operations, is an n^2 algorithm in the size of the domain.

7 Evidential Refinement

The combination of medium domain knowledge base and the medium evidential interpretation resulted in consistently better level-n values than the corresponding values under precise and highly imprecise evidential interpretations. These results indicate that there may be a degree of evidential imprecision that optimizes the identification process. To further investigate the effect of evidential interpretation on the level-n support and entropy values, two additional evidential interpretations were constructed. These evidential interpretations were constructed so that the U-uncertainty of the interpretations (the bell-shaped distributions) lie between the U-uncertainties of the medium and high distributions for the first two attributes. Table 7 shows the values for the α and β parameters for the bell-shaped distribution along with their corresponding U-uncertainty measures for the five evidential interpretations.

7.1 Level-n Support

The level-n support produced by the five evidential interpretations are summarized in Tables 8–15. These tables are given in pairs based on the reasoning paradigm employed. The first table in the pair has results using medium variance to generate observations and the second in the pair is with high variance.

For the sup-min technique, the best level-n results were obtained with the medium-2 evidential interpretation regardless of the precision in the domain knowledge base and the variance in observation generation. For the expected value technique, the best level-n results varied depending upon the precision of the domain knowledge base and the variance in the observation generation. With medium variance in the accuracy of the sensor data and imprecise domain data, the three medium evidential interpretations produced similar results. Each of these produced level-n support superior to both the high and precise evidential interpretations. Under the same conditions except with precise domain knowledge, the medium-2 evidential interpretation demonstrated superior discriminatory capabilities. The same pattern of behavior occurred when the expected value technique was used with highly inaccurate sensor data.

The probabilistic technique for medium variance and the precise domain knowledge had best level-n results with the medium evidential interpretation. For level-ns of 5 or more, the medium-1 interpretation provided the best results. Adding imprecision to the domain knowledge, the three medium evidential interpretations produced comparable results. With high variance, the medium-1 interpretation performed the best regardless of the imprecision in the domain knowledge base.

The optimum evidential interpretation for the Dempster-Shafer technique was also dependent on the imprecision in the domain knowledge base. For the medium vari-

Table 7: Evidential interpretations

Evidential Interpretation	Attribute 1			Attribute 2		
	α	β	U-uncertainty	α	β	U-uncertainty
Precise	0	1	0.00	0	1	0.00
Medium	5	10	3.89	0	2	0.79
Medium-1	10	10	4.25	1	2	1.58
Medium-2	5	15	4.29	0	3	1.40
High	10	15	4.63	1	3	1.95

ance and the precise domain knowledge base, the medium-2 evidential interpretation produced superior level-n values. For the medium precision domain knowledge base with medium variance the three medium interpretations and the high interpretation were comparable. This pattern also occurred with the high variance analysis.

Table 8: Level-n rankings for sup-min reasoning technique

	Sup-Min, Medium Variance									
	Evidential Interpretation									
	Precise		Medium		Medium-1		Medium-2		High	
n-level	P_{DB}	M_{DB}	P_{DB}	M_{DB}	P_{DB}	M_{DB}	P_{DB}	M_{DB}	P_{DB}	M_{DB}
1	1	44	45	23	28	14	54	37	33	14
2		61	64	42	51	39	78	64	61	39
3		72	74	54	70	64	89	74	81	64
4		78	80	66	76	77	95	84	88	78
5		80	82	72	82	85	96	89	94	86
6		81	83	78	85	93	98	95	97	93
7				85	86	96		98	97	96
8				92		97		99	98	97
9				94		98				98
10				95		99				99

Table 9: Level-n rankings for sup-min reasoning technique

	Sup-Min, High Variance									
	Evidential Interpretation									
	Precise		Medium		Medium-1		Medium-2		High	
n-level	P_{DB}	M_{DB}	P_{DB}	M_{DB}	P_{DB}	M_{DB}	P_{DB}	M_{DB}	P_{DB}	M_{DB}
1		8	9	13	5	13	18	20	13	9
2		15	16	31	16	37	39	43	33	28
3		16	17	44	19	58	44	63	43	56
4		17	18	50	21	67	49	77	49	60
5		17	18	59	21	71	52	79	52	67
6		18	19	66	22	74	54	84	54	72
7				74		81	54	91	54	75
8				74		81	54	91	54	78
9				75		81	54	91	54	80
10				76		82	55	92	55	81

Table 10: Level-n rankings for expected-value reasoning technique

	Expected-Value, Medium Variance									
	Evidential Interpretation									
	Precise		Medium		Medium-1		Medium-2		High	
n-level	P_{DB}	M_{DB}	P_{DB}	M_{DB}	P_{DB}	M_{DB}	P_{DB}	M_{DB}	P_{DB}	M_{DB}
1	1	44	39	53	28	50	30	51	14	44
2		61	65	76	51	75	62	76	43	71
3		72	76	90	70	90	76	90	61	88
4		78	81	95	76	95	92	95	71	93
5		80	81	97	82	97	95	96	81	96
6		81	83	99	85	99	96	99	90	99
7					86		97		94	
8							98		96	
9									98	
10										

Table 11: Level-n rankings for expected-value reasoning technique

	Expected-Value, High Variance									
	Evidential Interpretation									
	Precise		Medium		Medium-1		Medium-2		High	
n-level	P_{DB}	M_{DB}	P_{DB}	M_{DB}	P_{DB}	M_{DB}	P_{DB}	M_{DB}	P_{DB}	M_{DB}
1		8	9	22	5	21	19	20	11	20
2		15	17	44	16	44	33	46	26	45
3		16	18	55	19	58	44	67	35	67
4		17	19	64	21	67	51	77	44	77
5		17		67	21	71	53	83	51	83
6		18		73	22	78	55	90	53	90
7				75		81		93	54	94
8				75		81		93	54	94
9				75		81		94	54	94
10				76		82		95	55	95

Table 12: Level-n rankings for probabilistic reasoning technique

	Probabilistic, Medium Variance									
	Evidential Interpretation									
	Precise		Medium		Medium-1		Medium-2		High	
n-level	P_{DB}	M_{DB}	P_{DB}	M_{DB}	P_{DB}	M_{DB}	P_{DB}	M_{DB}	P_{DB}	M_{DB}
1	0	55	48	60	29	49	34	49	10	23
2		69	69	82	60	75	62	78	37	45
3		77	78	91	78	86	76	89	58	68
4		80	81	96	87	93	81	96	69	79
5		80	82	97	92	95	83	96	79	85
6		81	83	99	95	98	86	99	83	91
7					96	99			87	95
8					98				88	97
9									90	98
10										98

Table 13: Level-n rankings for probabilistic reasoning technique

	Probabilistic, High Variance									
	Evidential Interpretation									
	Precise		Medium		Medium-1		Medium-2		High	
n-level	P_{DB}	M_{DB}	P_{DB}	M_{DB}	P_{DB}	M_{DB}	P_{DB}	M_{DB}	P_{DB}	M_{DB}
1	0	10	13	33	20	35	6	32	11	22
2		15	18	51	39	52	17	53	25	42
3		17	18	62	48	67	18	63	34	54
4		17	19	69	52	79	20	67	35	67
5		18		72	53	83		71	37	72
6				74	54	88		75	38	74
7				75	54	90		76	39	78
8				76	54	92		77	40	82
9					54	92				83
10					55	93				84

Table 14: Level-n rankings for Dempster-Shafer reasoning technique

	Dempster-Shafer, Medium Variance									
	Evidential Interpretation									
	Precise		Medium		Medium-1		Medium-2		High	
n-level	P_{DB}	M_{DB}	P_{DB}	M_{DB}	P_{DB}	M_{DB}	P_{DB}	M_{DB}	P_{DB}	M_{DB}
1	1	50	55	66	28	62	61	61	33	62
2		67	69	85	51	86	83	86	61	82
3		76	79	96	70	96	91	96	81	95
4		80	81	97	76	97	97	97	88	96
5		80	82	98	82	98	98	98	94	98
6		81		99	85	99		99	97	99
7					86				97	
8									98	
9										
10										

Table 15: Level-n rankings for Dempster-Shafer reasoning technique

	Dempster-Shafer, High Variance									
	Evidential Interpretation									
	Precise		Medium		Medium-1		Medium-2		High	
n-level	P_{DB}	M_{DB}	P_{DB}	M_{DB}	P_{DB}	M_{DB}	P_{DB}	M_{DB}	P_{DB}	M_{DB}
1		10	12	33	5	33	19	35	15	37
2		16	17	54	16	54	29	65	36	63
3		17	18	62	19	70	37	78	47	80
4		18	19	68	21	74	39	85	49	85
5				72	21	78	40	89	52	87
6				75	22	80	42	93	55	93
7				76		82		94		94
8								95		95
9										
10										

7.2 Entropy Analysis

For the precise domain knowledge base, Table 16 gives the entropy measures for the medium-1 (M1) and medium-2 (M2) evidential interpretations along with the precise (P), medium (M) and high (H) interpretations. The first set of five rows are the results for the tests with the medium sensor accuracy and the second set is for high inaccuracy. Table 17 displays the entropy for the results using the medium domain knowledge base. The high imprecision domain knowledge base was not used because of the processing time required for the test runs.

For the precise domain knowledge (Table 16), the entropy values for the sup-min and expected value techniques are comparable for all the evidential interpretations. As the level of imprecision increases in the evidential interpretation so does the the entropy measure for these two techniques. This differs from the level-n findings in which the medium evidential interpretations produced superior results. Good level-n results accompanied by high entropy indicate that the selected object is receiving support, but that the breadth of the support distribution is large. That is, the fuzzy techniques are not effective in reducing support for fringe objects.

The results of the two probabilistic techniques provide additional support for the assertion that there may be an optimal evidential interpretation. Both the D-S and probabilistic method produce minimal entropy results with the medium evidential interpretations rather than with the precise or highly imprecise evidence.

Examining the level-n support and the entropy reinforces our observation that the effectiveness of identification process depends upon the precision in the evidential interpretation. Moreover, it is not highly precise data that enhances the procedure, but rather information with a sufficient degree of imprecision to ensure the capture of the selected object in the process. The imprecision needed varies with the support generation paradigm.

7.3 General Conclusions and Continuing Work

The report presents the results of an analysis of several paradigms for generating support in sensor-based evidential reasoning problem domains. Within the constraints of the simulations developed and tested by the GCT, expectation based probabilistic analysis, the Dempster-Shafer theory, and fuzzy techniques using expectation as a compatibility measure produced comparable results. There were, however, certain scenarios in which there was considerable difference among these techniques.

The independence of the evidence and the lack of correlation between the attributes in the domain descriptions incorporated into the GCT evaluation favor the techniques whose updating rule assumes these conditions. Further study should attempt to determine the effects of independence assumptions on the generation of

Table 16: Mean entropies and their standard deviations for Precise DB

Precise DB								
Interpretation	Sup-Min		Exp Value		Prob'c		D-S	
	$\overline{E}$	$S_{\overline{E}}$	$\overline{E}$	$S_{\overline{E}}$	$\overline{E}$	$S_{\overline{E}}$	$\overline{E}$	$S_{\overline{E}}$
P	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
M	0.86	0.77	0.84	0.76	0.83	0.78	0.57	0.61
M1	1.04	0.86	1.04	0.86	0.91	0.82	1.04	0.86
M2	1.73	0.78	1.78	0.80	1.84	0.75	0.94	0.69
H	1.91	0.79	1.94	0.80	1.58	0.84	1.28	0.79
P	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
M	0.85	0.70	0.82	0.70	0.61	0.68	0.51	0.59
M1	1.02	0.71	1.02	0.71	0.71	0.68	1.02	0.71
M2	1.47	0.80	1.47	0.82	1.29	0.84	0.95	0.66
H	1.57	0.84	1.58	0.85	1.12	0.95	1.10	0.78

Table 17: Mean entropies and their standard deviations for Medium DB

Medium DB								
Interpretation	Sup-Min		Exp Value		Prob'c		D-S	
	$\overline{E}$	$S_{\overline{E}}$	$\overline{E}$	$S_{\overline{E}}$	$\overline{E}$	$S_{\overline{E}}$	$\overline{E}$	$S_{\overline{E}}$
P	0.87	0.77	0.87	0.77	0.73	0.71	0.74	0.72
M	2.32	0.71	1.93	0.73	1.55	0.72	1.28	0.77
M1	2.43	0.68	2.19	0.70	2.23	0.63	1.49	0.77
M2	2.99	0.59	2.51	0.61	1.83	0.72	1.61	0.76
H	3.09	0.58	2.73	0.59	2.43	0.68	1.79	0.75
P	0.91	0.69	0.91	0.69	0.56	0.69	0.76	0.69
M	1.88	0.79	1.56	0.80	1.17	0.71	1.20	0.73
M1	1.95	0.78	1.78	0.78	1.90	0.72	1.39	0.71
M2	2.53	0.65	2.09	0.69	1.38	0.84	1.54	0.71
H	2.62	0.67	2.25	0.70	1.92	0.88	1.66	0.73

support.

The results indicated a strong relationship between the effectiveness of the reasoning paradigm and the precision of both the domain knowledge and the evidence. There appears to be an optimal combination of these two parameters that depends upon accuracy of the sensor. With the domain imprecision fixed, processing precise or highly imprecise evidence produces poorer results than an intermediate precision. Determination of a quantitative relation between these two parameters may enhance the effectiveness of the evidential support algorithms.

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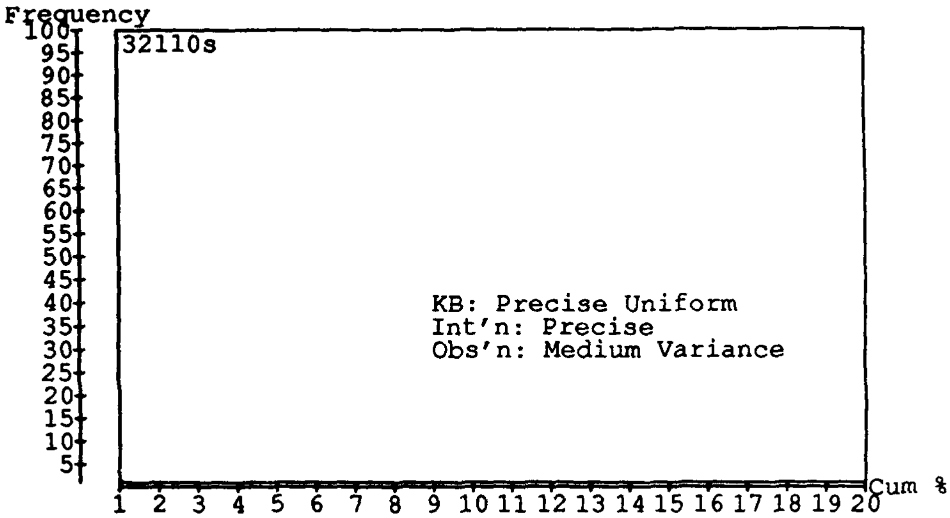
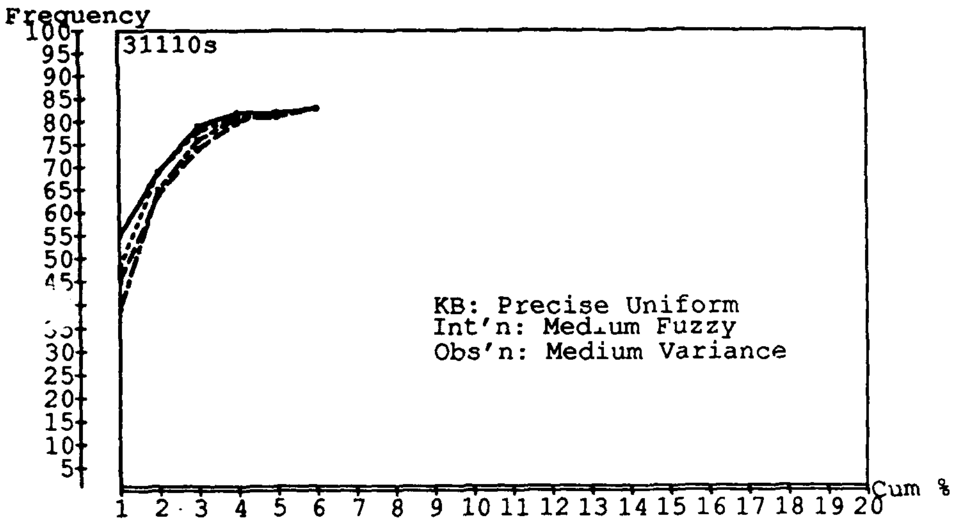
A Level-n Support Data

This appendix consists of graphs showing the level of support that resulted from the test runs discussed in Sections 5 and 6. The level of support *vs.* its frequency is plotted for each reasoning method. Each graph on the page shows the performance of the four reasoning methods. The Dempster-Shafer curve is shown by a solid line, the probabilistic curve by a dotted line, the fuzzy evaluation using sup-min composition by a dashed line, and the fuzzy evaluation using the expected-value operator by a dot-and-dashed line. The scenario represented by a graph is indicated by the number to its left. These numbers refer to the descriptions in Table 1. There is also a description on the graph itself. In graphs 9, 12, 15, and 24, with high imprecision in both the domain data and the evidential interpretation, the Dempster-Shafer data were not collected due to the CPU time required by the test run.

Each page contains three graphs with the same domain knowledge base. The three graphs are obtained from varying the evidential precision. The top graph gives the results when using the precise evidential interpretation. The middle graph uses the medium evidential interpretation. The bottom graph uses the high evidential interpretation. The first five pages give the results using the medium variance for the sensor accuracy. In the remaining five pages, the graphs are from test runs with high variance.


Frequency
32110s
KB: Precise Uniform
Int'n: Precise
Obs'n: Medium Variance
Cum %

1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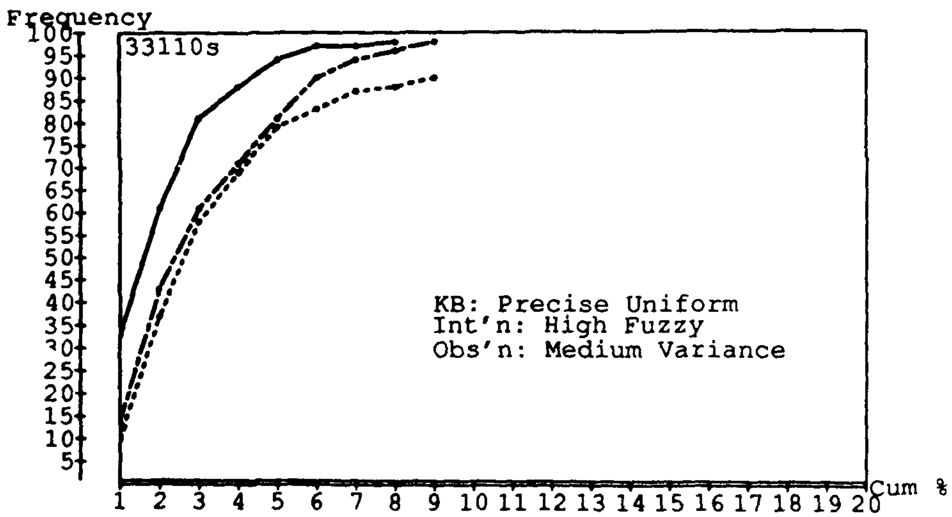
Int'n: Medium Fuzzy
Obs'n: Medium Variance
Cum %

2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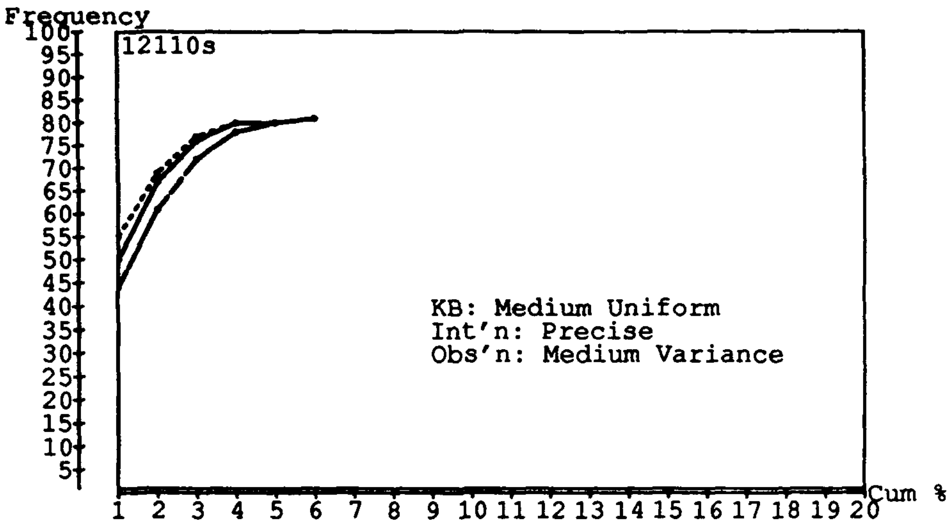
KB: Precise Uniform
Int'n: High Fuzzy
Obs'n: Medium Variance
Cum %

3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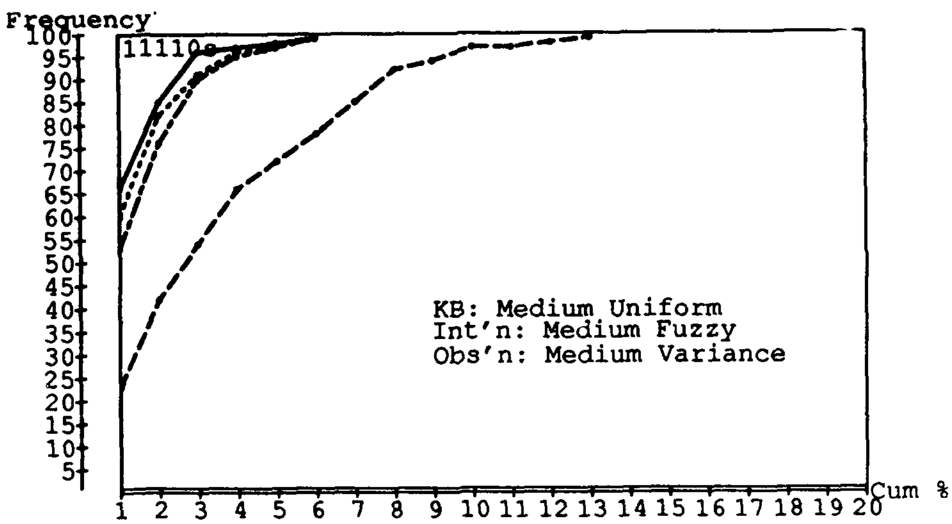
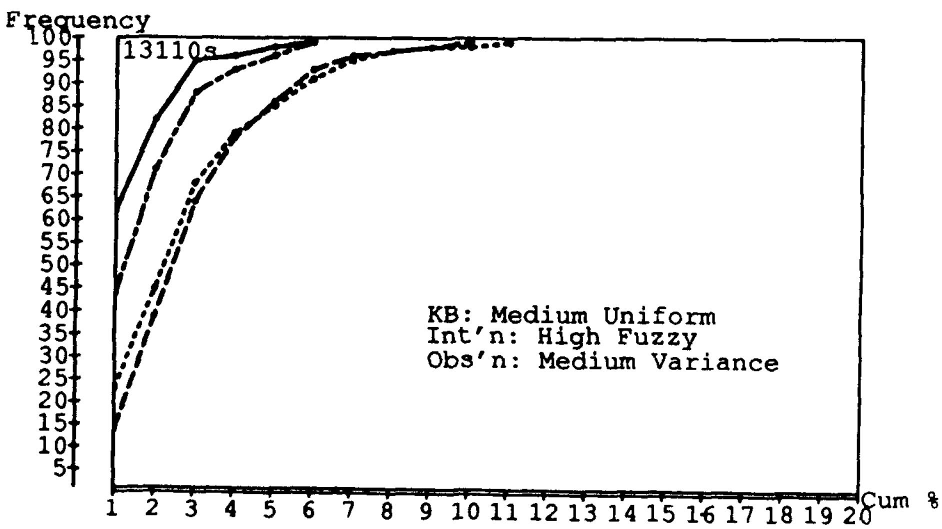
Int'n: Precise
Obs'n: Medium Variance
Cum %

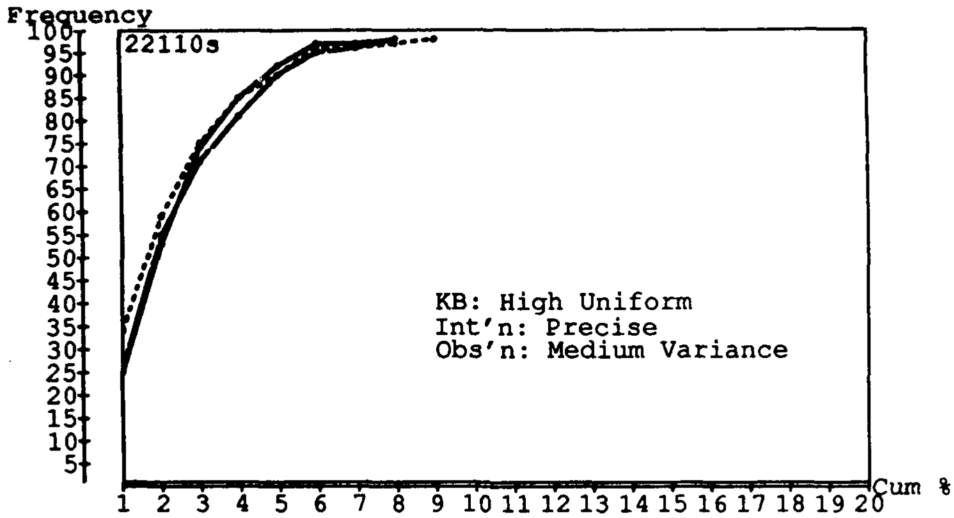
4


Frequency
11110s
KB: Medium Uniform
Int'n: Medium Fuzzy
Obs'n: Medium Variance
Cum %

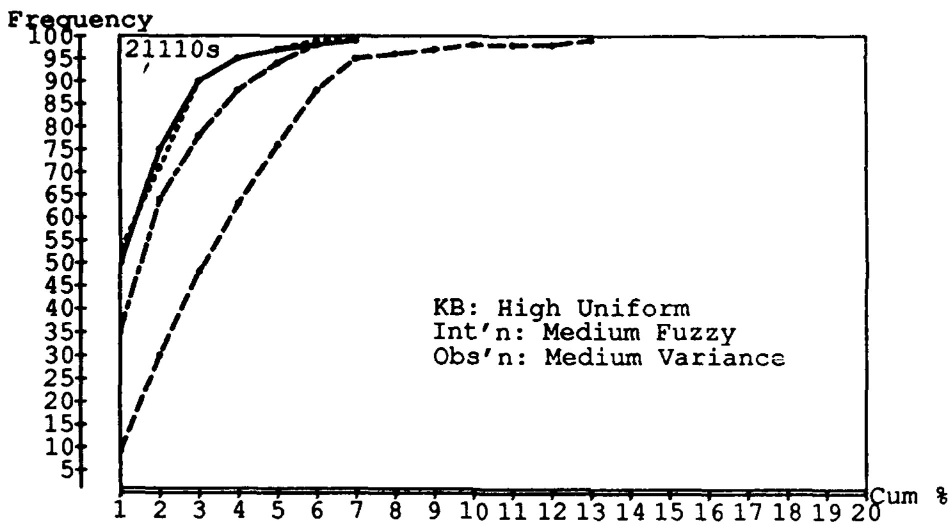
5


Frequency
13110s
KB: Medium Uniform
Int'n: High Fuzzy
Obs'n: Medium Variance
Cum %

6


Frequency
100
95
90
85
80
75
70
65
60
55
50
45
40
35
30
25
20
15
10
5
22110s
KB: High Uniform
Int'n: Precise
Obs'n: Medium Variance
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Cum %

7


Frequency
100
95
90
85
80
75
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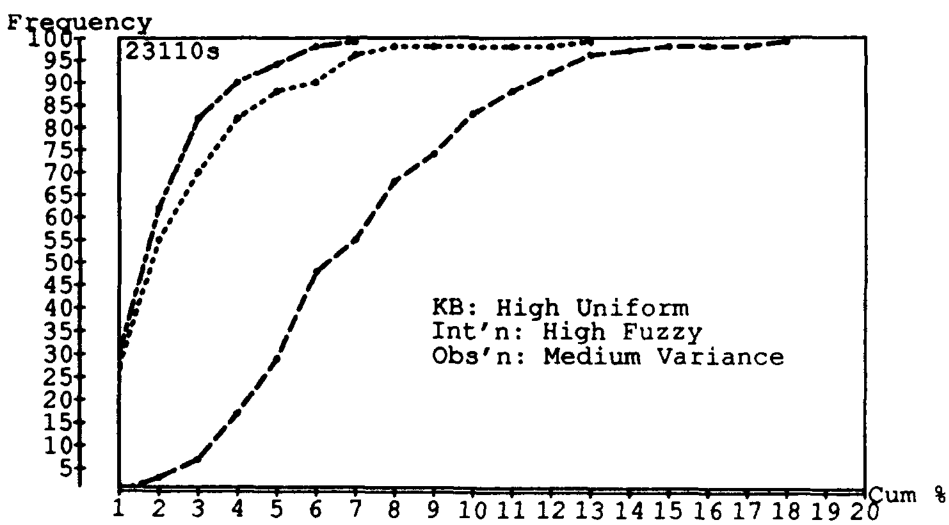
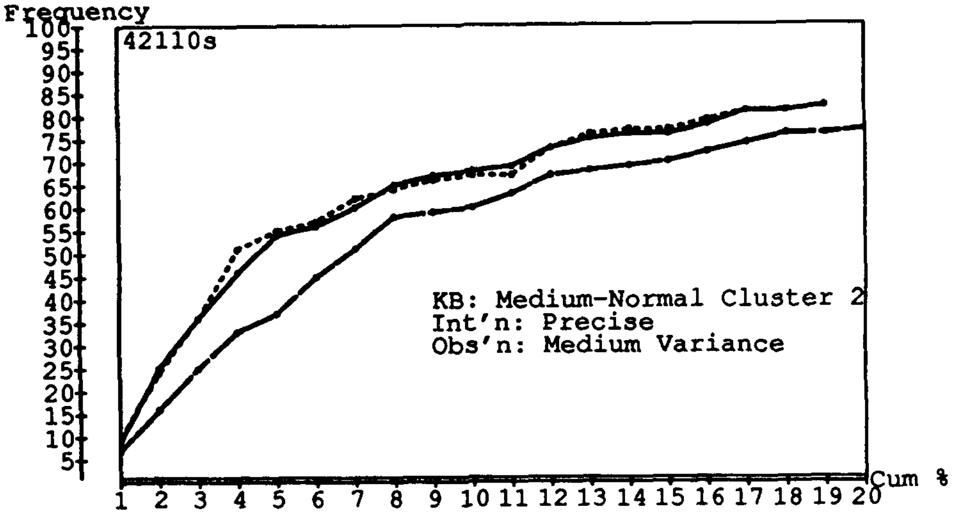
65
60
55
50
45
40
35
30
25
20
15
10
5
21110s
KB: High Uniform
Int'n: Medium Fuzzy
Obs'n: Medium Variance
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Cum %

8


Frequency
100
95
90
85
80
75
70
65
60
55
50
45
40
35
30
25
20
15
10
5
23110s
KB: High Uniform
Int'n: High Fuzzy
Obs'n: Medium Variance
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Cum %

9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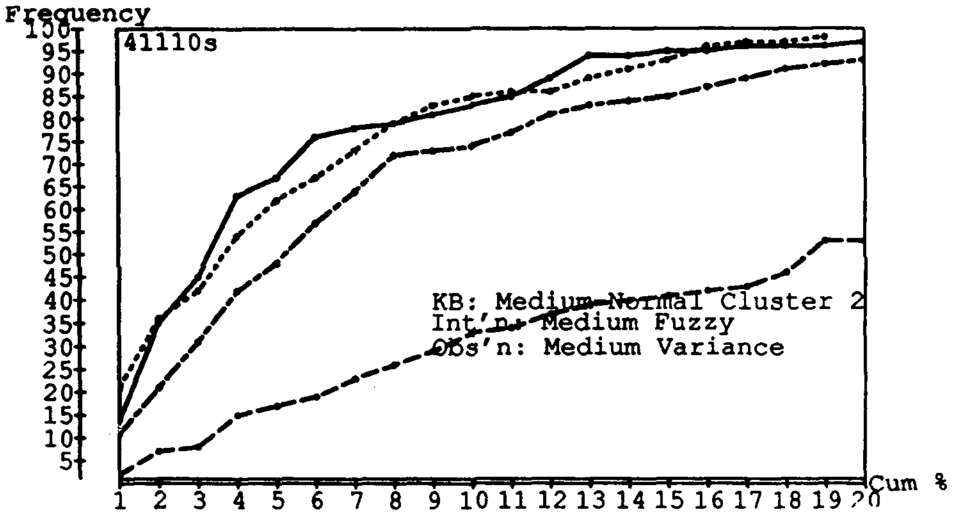
Int'n: Precise
Obs'n: Medium Variance
Cum %

10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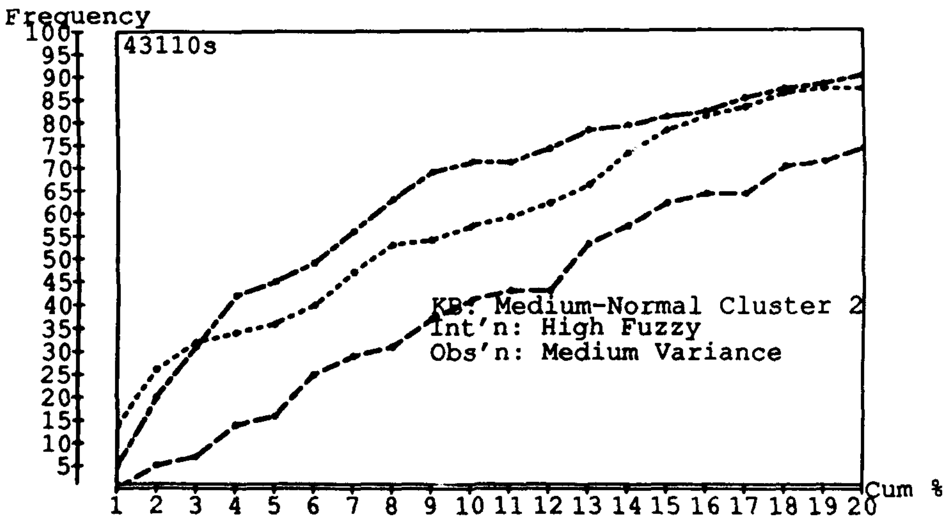
Obs'n: Medium Variance
Cum %

11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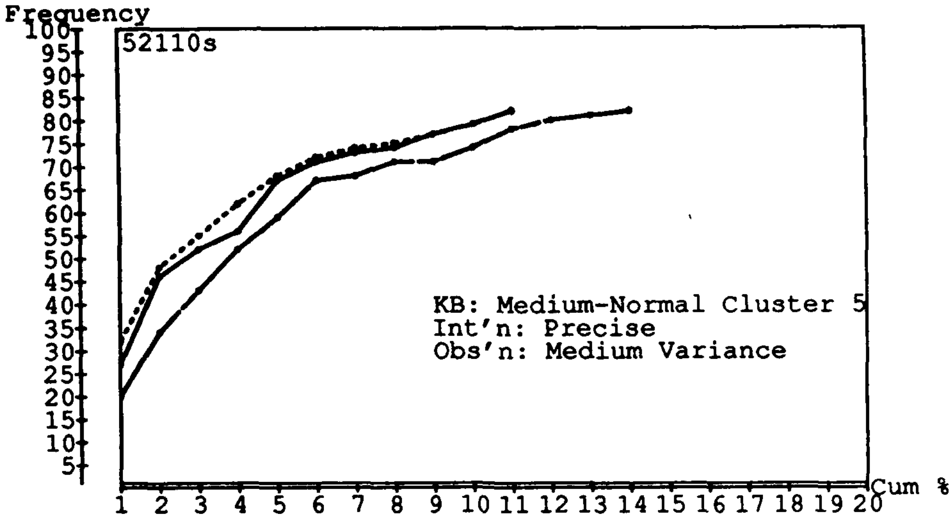
Int'n: High Fuzzy
Obs'n: Medium Variance
Cum %

12


Frequency
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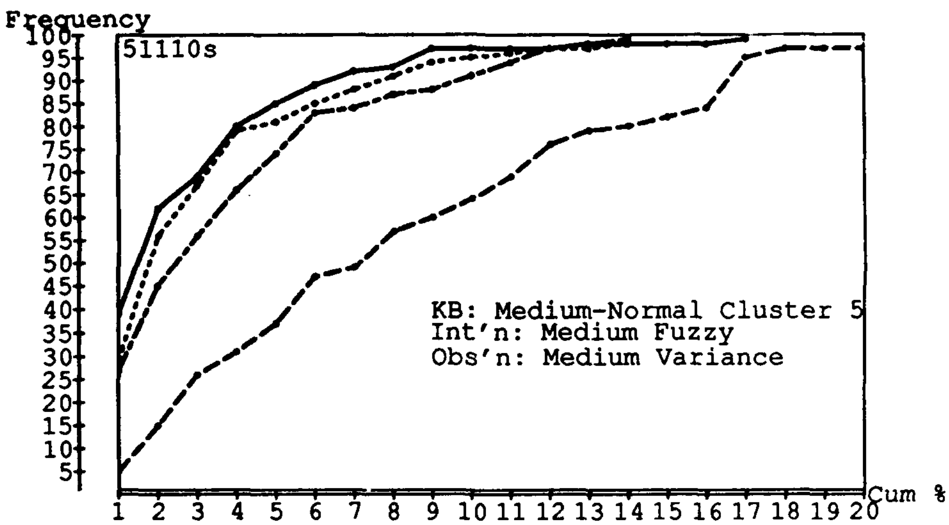
KB: Medium-Normal Cluster 5
Int'n: Precise
Obs'n: Medium Variance
Cum %

13


Frequency
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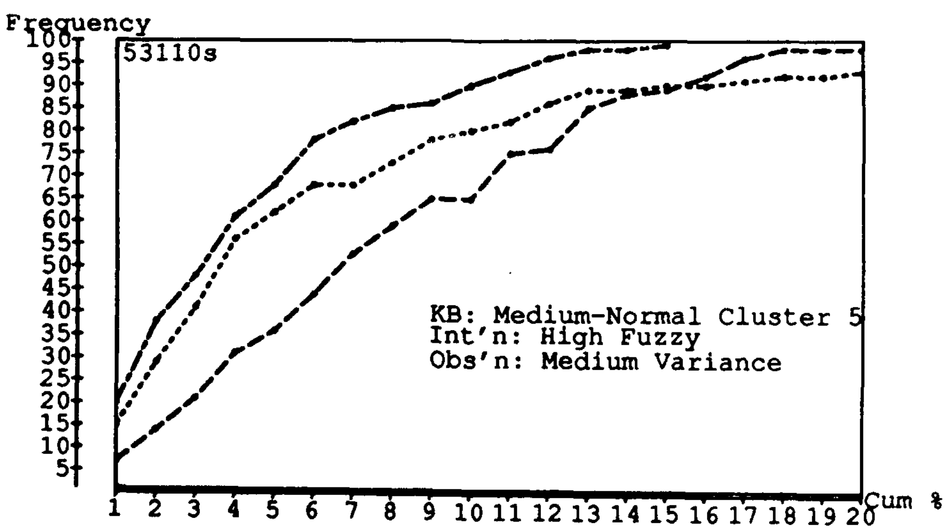
KB: Medium-Normal Cluster 5
Int'n: Medium Fuzzy
Obs'n: Medium Variance
Cum %

14


Frequency
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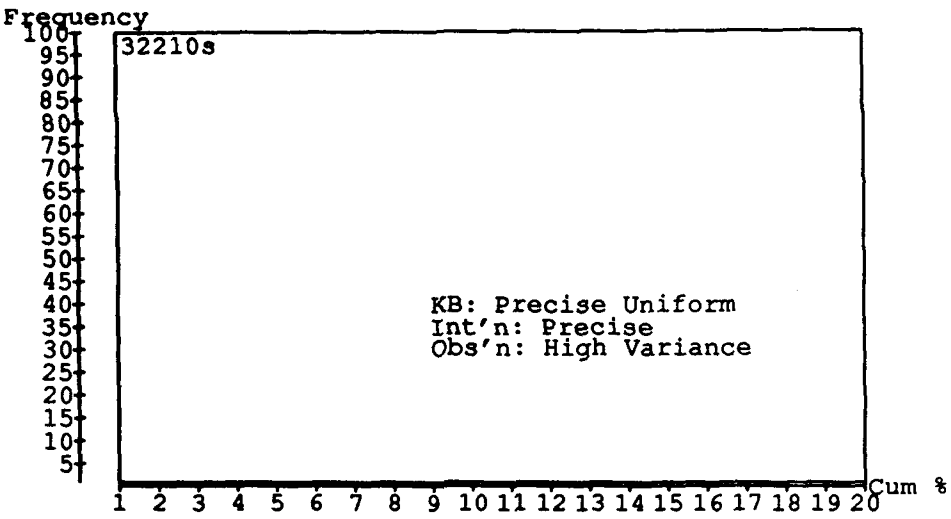
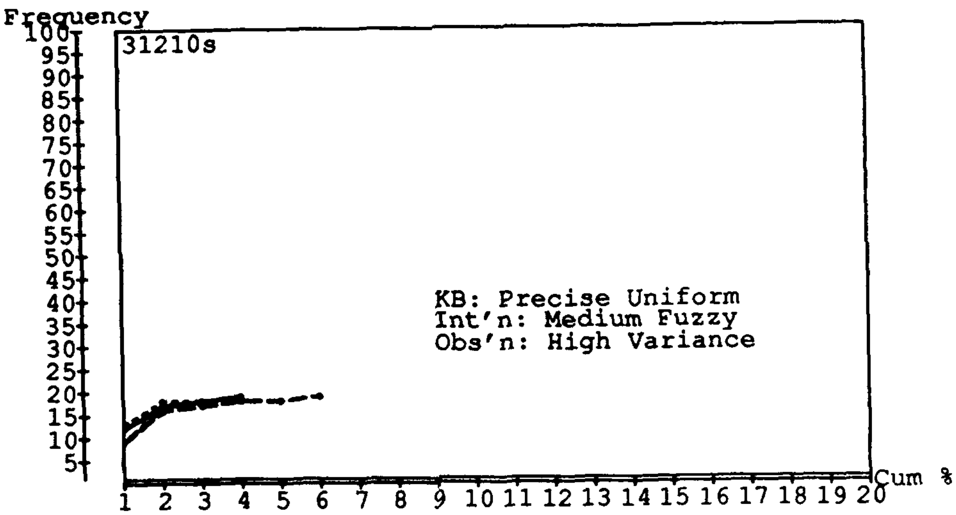
Int'n: High Fuzzy
Obs'n: Medium Variance
Cum %

15



Frequency
32210s
KB: Precise Uniform
Int'n: Precise
Obs'n: High Variance
Cum %

16



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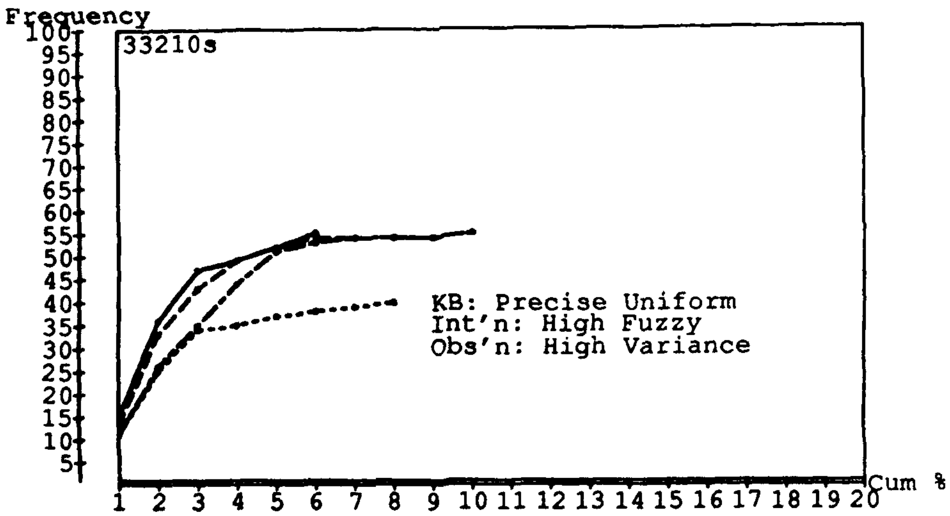
Int'n: Medium Fuzzy
Obs'n: High Variance
Cum %

17



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Obs'n: High Variance
Cum %

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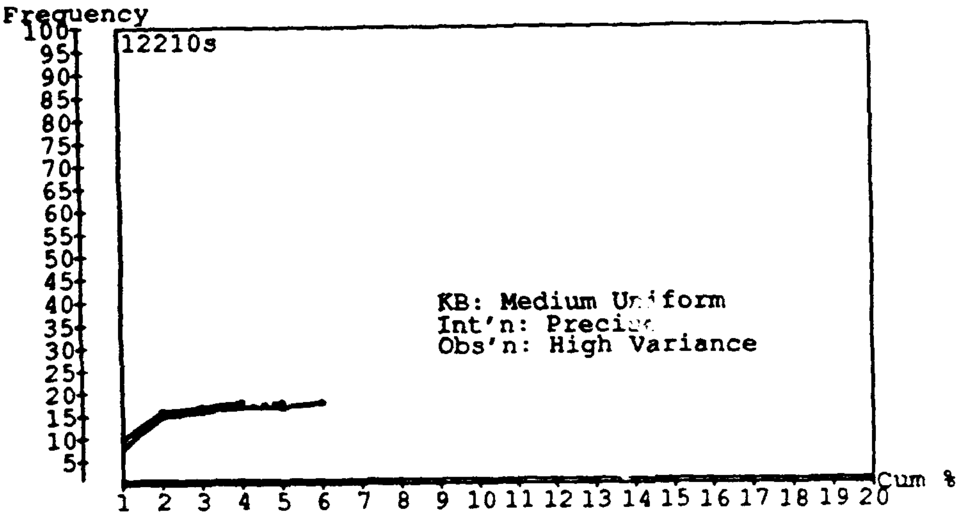
Frequency
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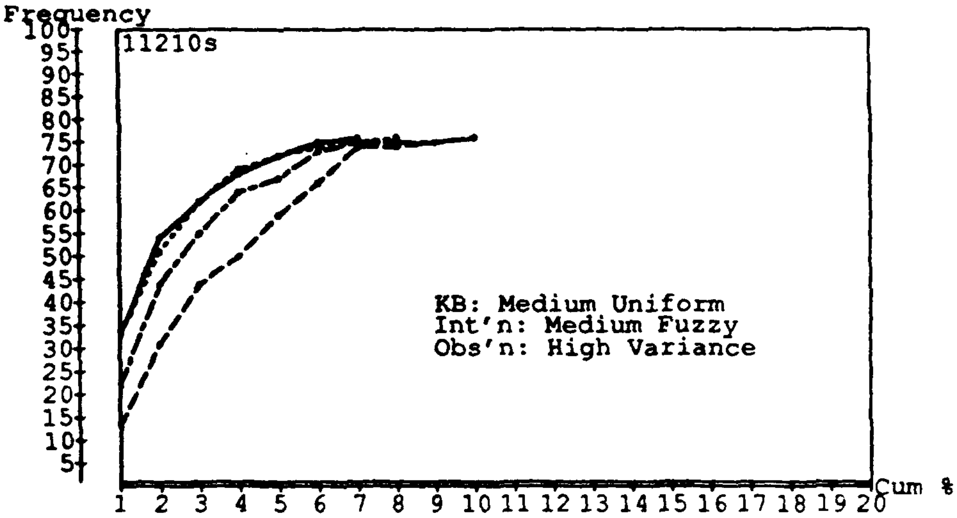
Obs'n: High Variance
Cum %

19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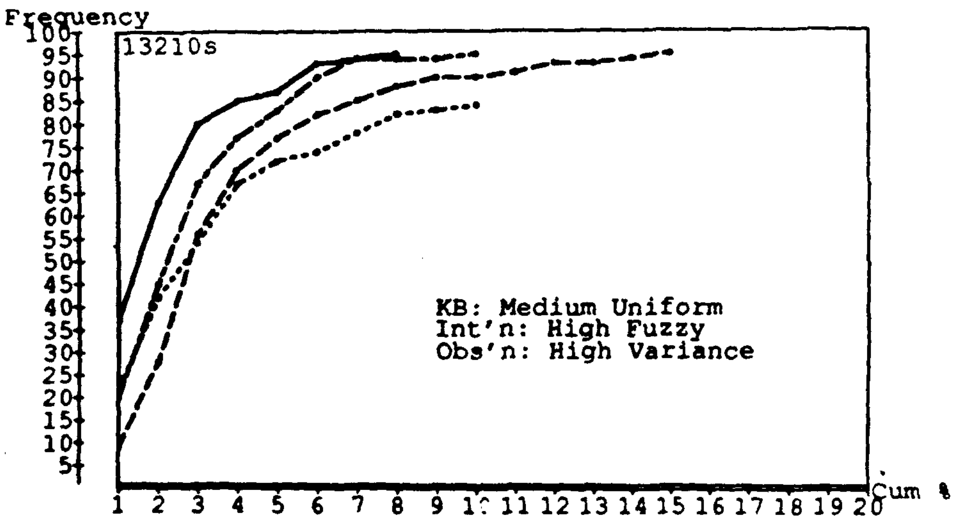
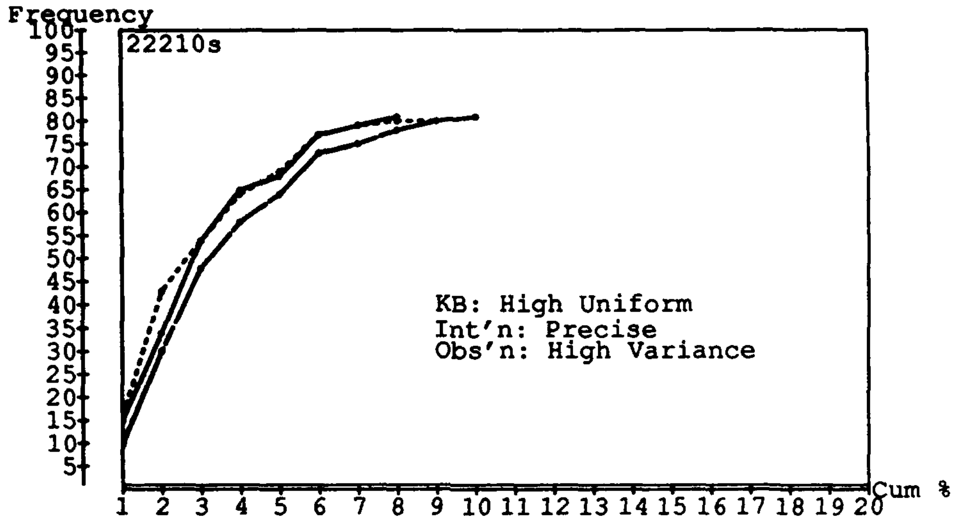
Int'n: Medium Fuzzy
Obs'n: High Variance
Cum %

20


Frequency
13210s
KB: Medium Uniform
Int'n: High Fuzzy
Obs'n: High Variance
Cum %

21


Frequency
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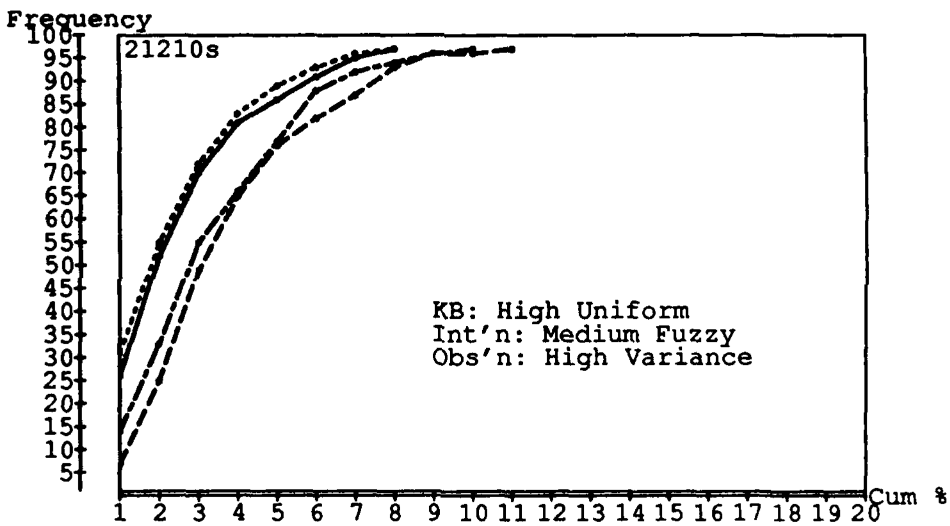
KB: High Uniform
Int'n: Precise
Obs'n: High Variance
Cum %

22


Frequency
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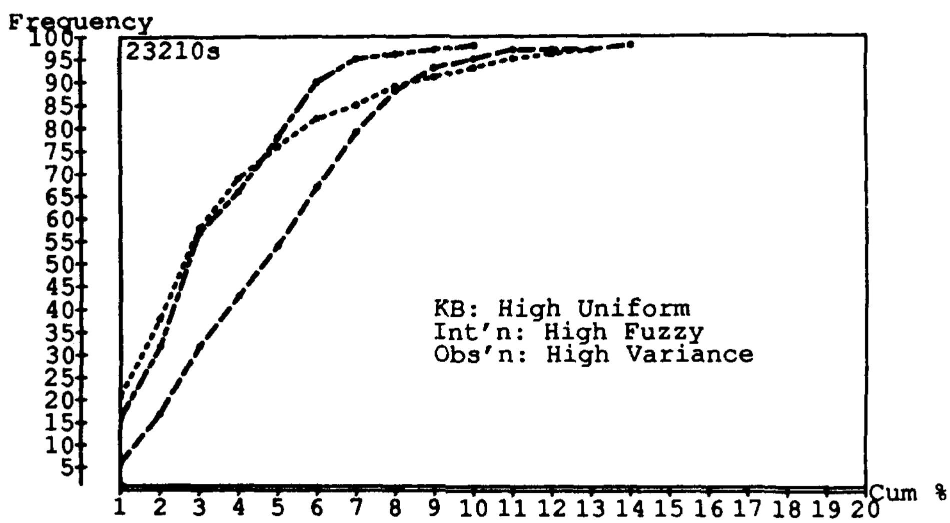
KB: High Uniform
Int'n: Medium Fuzzy
Obs'n: High Variance
Cum %

23


Frequency
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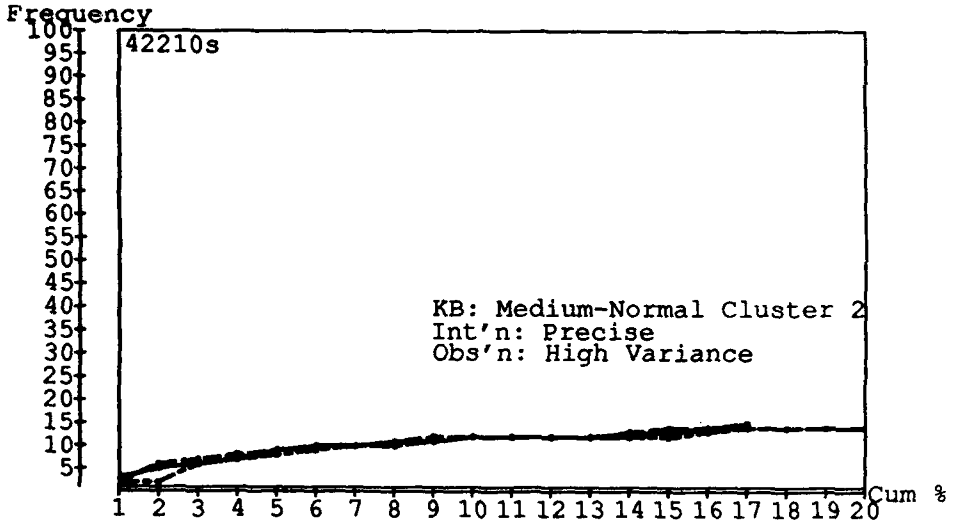
Int'n: High Fuzzy
Obs'n: High Variance
Cum %

24



Frequency
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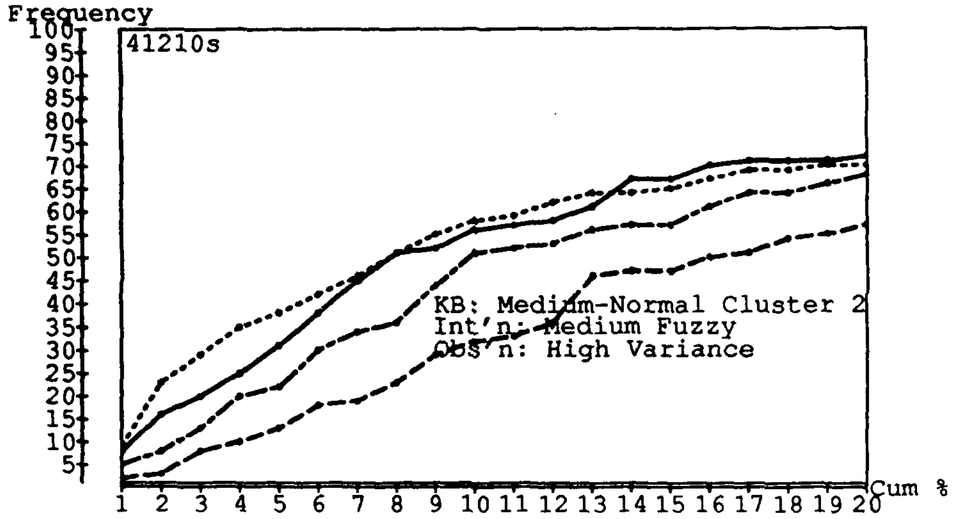
KB: Medium-Normal Cluster 2
Int'n: Precise
Obs'n: High Variance
Cum %

25



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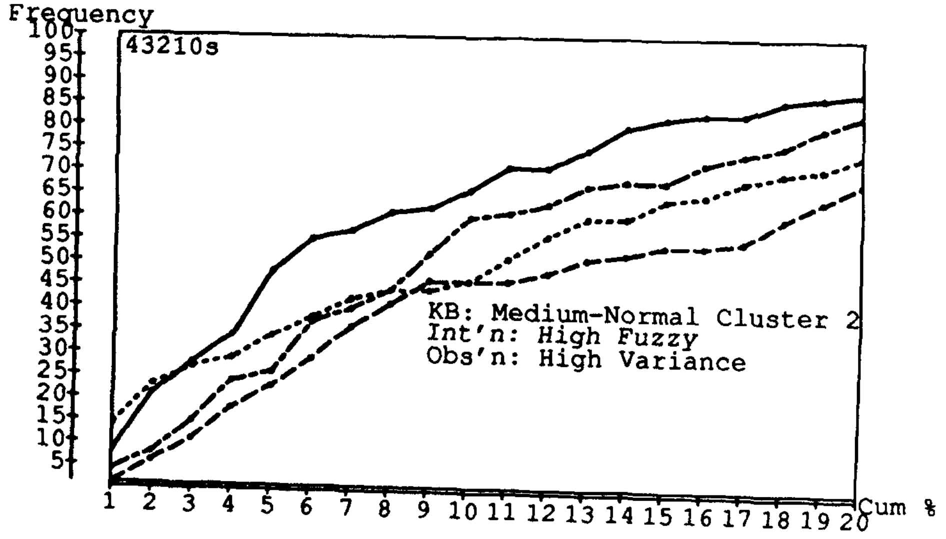
Int'n: Medium Fuzzy
Obs'n: High Variance
Cum %

26



Frequency
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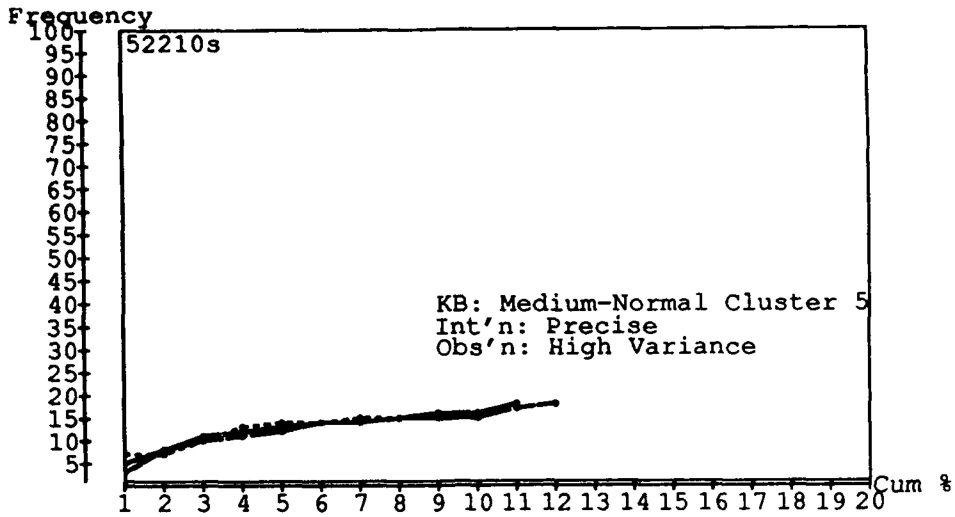
KB: Medium-Normal Cluster 2
Int'n: High Fuzzy
Obs'n: High Variance
Cum %

27


Frequency
52210s
KB: Medium-Normal Cluster 5
Int'n: Precise
Obs'n: High Variance
Cum %
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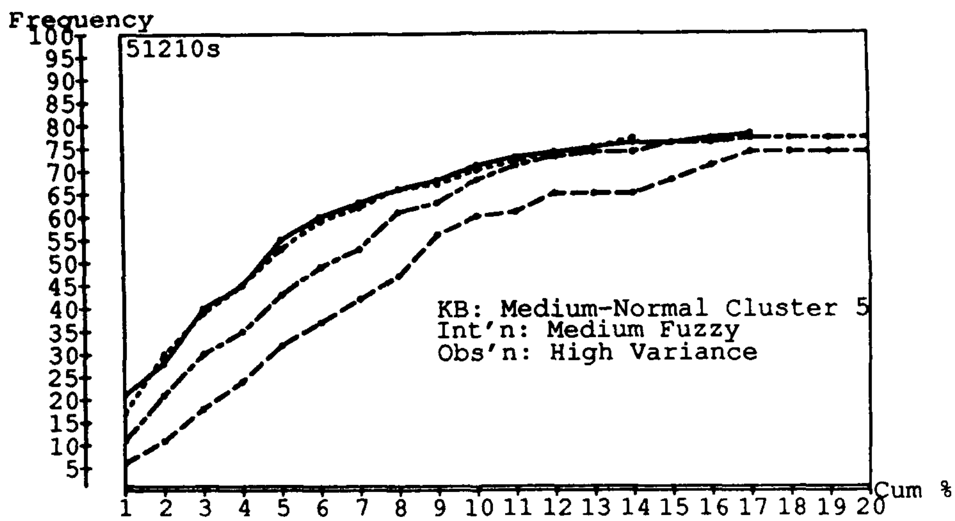
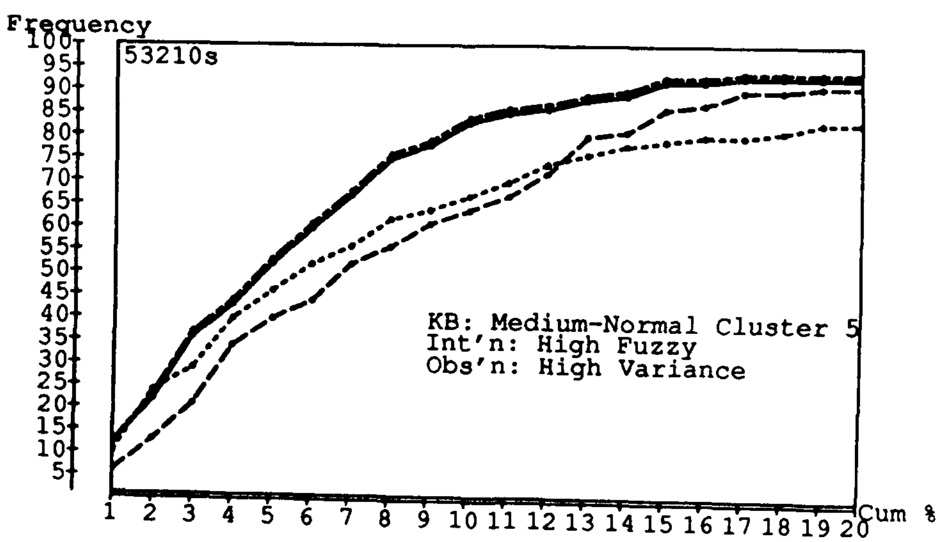
28


Frequency
51210s
KB: Medium-Normal Cluster 5
Int'n: Medium Fuzzy
Obs'n: High Variance
Cum %
100 95 90 85 80 75 70 65 60 55 50 45 40 35 30 25 20 15 10 5
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

29


Frequency
53210s
KB: Medium-Normal Cluster 5
Int'n: High Fuzzy
Obs'n: High Variance
Cum %
100 95 90 85 80 75 70 65 60 55 50 45 40 35 30 25 20 15 10 5
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

30

B Radar Knowledge Base

The frame for the representation of emitters was given in Example 7. The emitter knowledge base consists of the following six emitters. These have been provided for testing purposes only and do not portray actual emitters.

Emitter type 1

Radar Frequency (RF) Details

(a) The possible limits are 2.30 - 3.47 GHz.
The probable operating limits are 2.54 - 3.24 GHz.
The most observed band limits are 2.77 - 3.00 GHz.

(b) The RF is nominally constant.

Pulse Repetition Interval (PRI) Details

(a) The radar operates in either a constant PRI mode or a 2-element, 2-position stagger mode.

(b) In constant PRI mode, PRI limits are
   (i) 90.96 - 101.27 μsec
   (ii) 116.87 - 133.62 μsec
   (iii) 179.19 - 199.22 μsec

(c) In staggered mode, the PRI elements are
   (i) element 1: 66.42 - 71.11 μsec
   (ii) element 2: 136.36 - 157.57 μsec
   or
   (i) element 1: 74.76 - 107.90 μsec
   (ii) element 2: 191.08 - 194.09 μsec

Pulse Width (PW) Details

(a) The limits are 0.2 - 0.3 μsec.

Scan Details

(a) The scan is either conical (CON) or bidirectional sector (BDS).

(b) The conical scan period is between 0.04 and 0.06 sec.

(c) The bidirectional scan period lies between 2.0 and 4.0 sec.

Emitter type 2

Radar Frequency (RF) Details

(a) The possible limits are 2.38 - 2.62 GHz.
The probable band is 2.43 - 2.57 GHz.
The most likely band is 2.48 - 2.52 GHz.

(b) Manual tuning of RF to any value within range is possible. Transmitter is usually blanked for 10 to 20 seconds at this time but has been reported to be free running.

Pulse Repetition Interval (PRI) Details

(a) The PRI is nominally constant.

(b) There are five ranges as follows:
   (i) 77.72 - 84.15 μsec
   (ii) 89.98 - 119.53 μsec
   (iii) 120.03 - 120.85 μsec
   (iv) 138.61 - 150.07 μsec
   (v) 170.05 - 179.91 μsec

Pulse Width (PW) Details

(a) The limits are 0.2 - 0.5 μsec.

Scan Details

(a) The scan is unidirectional sector (UDS).

(b) The scan period is between 0.5 and 1.5 sec.

Emitter type 3

Radar Frequency (RF) Details

(a) The possible limits are 2.24 - 3.71 GHz.
The probable band is 2.54 - 3.42 GHz.
The most likely band is 2.83 - 3.13 GHz.

(b) The RF is nominally constant.

(b) Frequency separation among co-located emitters is typically 30 to 80 MHz.

Pulse Repetition Interval (PRI) Details

(a) The PRI is nominally constant.

(b) The PRI limits are divided into three ranges as follows
   - (i) 55.39 - 88.47 μsec
   - (ii) 125.03 - 130.28 μsec
   - (iii) 164.19 - 173.20 μsec

Pulse Width (PW) Details

(a) The limits are 0.3 - 0.8 μsec.

Scan Details

(a) The scan is bidirectional sector (BDS).

(b) The conical scan period is between 0.04 and 0.06 sec.

(c) The scan period lies between 0.2 and 0.7 sec.

Emitter type 4

Radar Frequency (RF) Details
(a) The possible limits are 2.08 - 2.46 GHz.
The most likely band is 2.23 - 2.30 GHz.
(b) The RF is nominally constant.

Pulse Repetition Interval (PRI) Details
(a) The PRI is nominally constant.
(b) The PRI limits are divided into four ranges as follows:
- (i) 79.96 - 97.92 μsec
- (ii) 133.79 - 143.40 μsec
- (iii) 150.81 - 181.63 μsec
- (iv) 212.04 - 223.32 μsec

Pulse Width (PW) Details
(a) The limits are 0.5 - 1.0 μsec.

Scan Details
(a) The scan is either unidirectional sector (UDS) or conical (CON).
(b) For both types, the scan period is between 0.05 and 0.06 sec.

Emitter type 5

Radar Frequency (RF) Details

(a) The possible limits are 2.81 - 3.97 GHz.
The probable operating limits are 3.04 - 3.74 GHz.
The most likely band is 3.27 - 3.51 GHz.

(b) Switching among three preset values nominally 20 to 50 MHz apart is possible. Switching time is reported to be 1 to 2 sec. Transmitter is blanked during switching.

Pulse Repetition Interval (PRI) Details

(a) The radar operates in a 2-element, 2-position stagger mode.

(b) The PRI elements are
   (i) element 1: 53.25 - 63.34 μsec
   (ii) element 2: 137.70 - 156.55 μsec
   or
   (i) element 1: 117.04 - 122.80 μsec
   (ii) element 2: 63.70 - 110.12 μsec

Pulse Width (PW) Details

(a) The limits are 0.3 - 0.7 μsec.

Scan Details

(a) The scan is conical (CON).

(b) The scan period is between 0.05 and 0.01 sec.

Emitter type 6

Radar Frequency (RF) Details
(a) The possible limits are 2.55 - 3.27 GHz.
(b) The RF is nominally constant.

Pulse Repetition Interval (PRI) Details
(a) The PRI is normally staggered but may be fixed.
(b) The PRI limits are 100.00-392.00 μsec.
(c) In staggered mode, the difference between elements is 3.47 μsec.

Pulse Width (PW) Details
(a) The limits are 0.15 - 0.35 μsec.

Scan Details
(a) The scan is bidirectional sector (BDS).
(b) The scan period is between 5 and 10 sec.
(c) The bidirectional scan period lies between 2.0 and 4.0 sec.

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